F1 2026 LS S2 riešenie

1.

$$x(t) = \frac{v_0}{2}\left(t + \frac{\tau}{\pi}\sin\frac{\pi t}{\tau}\right)$$

$$v(t) = \frac{dx}{dt} = \frac{v_0}{2}\left(1 + \frac{\tau}{\pi}\cdot\frac{\pi}{\tau}\cos\frac{\pi t}{\tau}\right)$$

$$v(t) = \frac{v_0}{2}\left(1 + \cos\frac{\pi}{\tau}t\right)$$

$$a(t) = \frac{dv}{dt} = \frac{v_0}{2}\left(-\frac{\pi}{\tau}\sin\frac{\pi}{\tau}t\right)$$

$$a(t) = -\frac{\pi}{2}\frac{v_0}{\tau}\sin\frac{\pi t}{\tau}$$

$$t = \tau \Rightarrow v(\tau) = \frac{v_0}{2}(1+\cos\pi) = \frac{v_0}{2}(1-1) = 0$$

$$\vec{F} = m\vec{a} \Rightarrow |F_{max}| \sim |a_{max}|$$

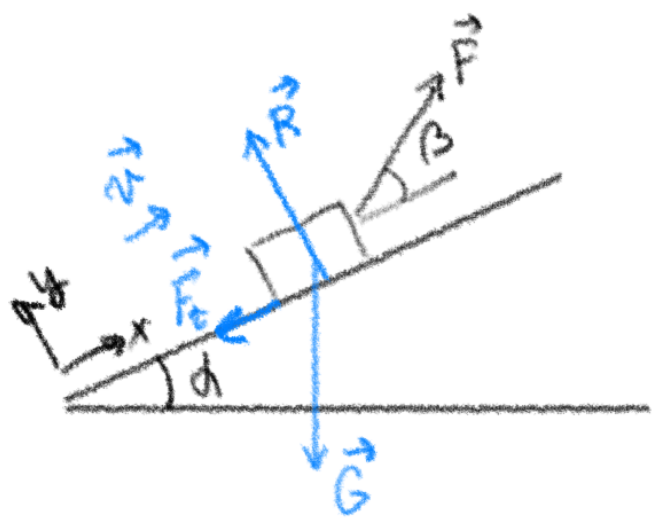
$$a_{max}: \sin\frac{\pi t}{\tau} = 1 \Rightarrow \frac{\pi t_m}{\tau} = \frac{\pi}{2} \Rightarrow t_m = \frac{\tau}{2}$$

$$|a_{max}| = \frac{\pi}{2}\frac{v_0}{\tau}\sin\frac{\pi}{2} = \frac{\pi}{2}\frac{v_0}{\tau}$$

2. $m_1, \mu_1, \alpha_1, \beta$

F_j

$$\vec{F} + \vec{G} + \vec{R} + \vec{F}_t = \vec{0}$$



x: $F\cos\beta - G\sin\alpha - F_t = 0$

y: $F\sin\beta - G\cos\alpha + R = 0$

$$F_t = \mu R$$

$$F\cos\beta - mg\sin\alpha - \mu R = 0$$
$$F\sin\beta - mg\cos\alpha + R = 0 \Rightarrow R = mg\cos\alpha - F\sin\beta$$

$$F\cos\beta - mg\sin\alpha - \mu(mg\cos\alpha - F\sin\beta) = 0$$

$$F(\cos\beta + \mu\sin\beta) - mg(\sin\alpha + \mu\cos\alpha) = 0$$

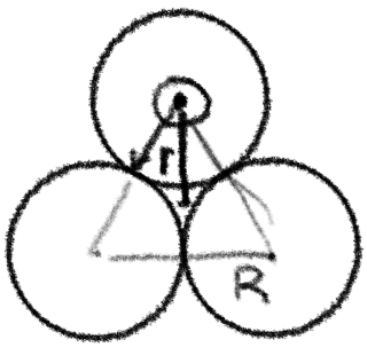
$$F = mg\,\frac{\sin\alpha + \mu\cos\alpha}{\cos\beta + \mu\sin\beta}$$

3. R; $J_T = \frac{1}{2} m_1 R^2$

T;

$$\omega = \sqrt{\frac{mgr}{J}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{J}{mgr}}$$

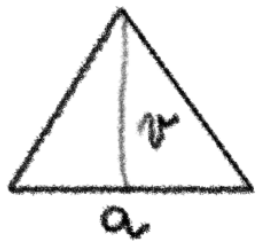


$$J = 3 \cdot \frac{1}{2} m_1 R^2 + 2 m_1 (2R)^2 =$$

$$= \frac{3}{2} mR^2 + 8 m_1 R^2 = \frac{19}{2} m_1 R^2$$

$$m_1 = \frac{m}{3} \Rightarrow J = \frac{19}{2}\frac{m}{3} R^2 = \frac{19}{6} mR^2$$

r: $\frac{2}{3}$ z výšky $\triangle$



$$v^2 + \frac{a^2}{4} = a^2 \Rightarrow v^2 = a^2 - \frac{a^2}{4} = \frac{3}{4} a^2$$

$$v = \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{2} 2R = \sqrt{3} R$$

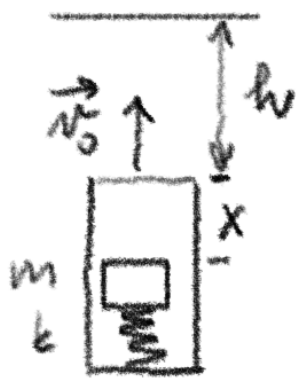
$$r = \frac{2}{3}\sqrt{3} R = \frac{2}{\sqrt{3}} R$$

$$T = 2\pi \sqrt{\frac{19 m R^2 \sqrt{3}}{6 m g \cdot 2R}} = 2\pi \sqrt{\frac{19\sqrt{3}}{12}\frac{R}{g}}$$

4. W, k, m

h;

ZZME



$$W = mgx + \frac{1}{2} m v_0^2$$

$$\frac{1}{2} m v_0^2 = mgh \Rightarrow h = \frac{1}{2}\frac{v_0^2}{g}$$

$$W = \frac{1}{2} k x^2 \Rightarrow x = \sqrt{\frac{2W}{k}}$$

$$W = mg\sqrt{\frac{2W}{k}} + \frac{1}{2} m v_0^2 \Rightarrow v_0^2 = \frac{2W}{m} - 2g\sqrt{\frac{2W}{k}}$$

$$h = \frac{1}{2g}\left(\frac{2W}{m} - 2g\sqrt{\frac{2W}{k}}\right) = \frac{W}{gm} - \sqrt{\frac{2W}{k}}$$

5. n_1, V_{11}, T_1, p_2

Q_j

izotermický dej: $\Delta U = 0$

$Q = W \qquad p_1 V_1 = p_2 V_2$

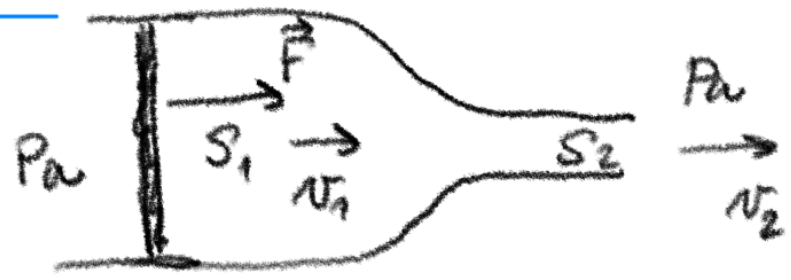
$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{p_1 V_1}{V}\,dV = p_1 V_1 [\ln V]_{V_1}^{V_2} = p_1 V_1 \ln \frac{V_2}{V_1}$$

$$pV = nRT \quad \Rightarrow V_2 = \frac{nRT}{p_2} \Rightarrow \frac{V_2}{V_1} = \frac{nRT}{p_2 V_1}$$

$$W = nRT \ln \frac{nRT}{p_2 V_1} = Q > 0$$

6. $S_{11}, \varsigma_1, F_1, S_2 << S_1$

v_{11}, v_2



$$p_1 + \frac{1}{2}\varsigma v_1^2 = p_a + \frac{1}{2}\varsigma v_2^2$$

$$S_1 v_1 = S_2 v_2 \quad \Rightarrow \quad v_2 = \frac{S_1}{S_2} v_1$$

$$p_1 = p_a + \frac{F}{S_1}$$

$$\frac{F}{S_1} + \frac{1}{2}\varsigma v_1^2 = \frac{1}{2}\varsigma v_2^2$$

$$\frac{F}{S_1} + \frac{1}{2}\varsigma v_1^2 = \frac{1}{2}\varsigma \frac{S_1^2}{S_2^2} v_1^2$$

$$\frac{1}{2}\varsigma v_1^2 \left(\frac{S_1^2}{S_2^2} - 1\right) = \frac{F}{S_1}$$

$$v_1 = \sqrt{\frac{2F}{S_1 \varsigma} \cdot \frac{S_2^2}{S_1^2 - S_2^2}}$$

$$v_2 = \frac{S_1}{S_2} v_1 = \sqrt{\frac{2F}{S_1 \varsigma} \frac{S_1^2}{S_1^2 - S_2^2}}$$