F1 2026 LS Skúška 1 – riešenie

1.

$v_x = A$

$v_y = B - 2Ct$; $A, B, C > 0$

$\vec{r}_0 = x_0\vec{i} + y_0\vec{j}$

$x(t), y(t)$; t_1 ; $\vec{r}(t_1)$; d

$$x = \int v_x\,dt = \int A\,dt = At + K_1$$

$x(0) = K_1 = x_0 \qquad x(t) = x_0 + At$

$$y = \int v_y\,dt = \int (B - 2Ct)\,dt = Bt - Ct^2 + K_2$$

$y(0) = K_2 = y_0 \qquad y(t) = y_0 + Bt - Ct^2$

$y(t_1) = y_m \rightarrow \frac{dy}{dt} = 0 \Leftrightarrow v_y = 0$

$v_y(t_1) = B - 2Ct_1 = 0 \Rightarrow t_1 = \frac{B}{2C}$

$x(t_1) = x_0 + At_1 = x_0 + \frac{AB}{2C}$

$y(t_1) = y_0 + Bt_1 - Ct_1^2 = y_0 + \frac{B^2}{2C} - C\frac{B^2}{4C^2} = y_0 + \frac{B^2}{4C}$

$\vec{r}(t_1) = \left(x_0 + \frac{AB}{2C}\right)\vec{i} + \left(y_0 + \frac{B^2}{4C}\right)\vec{j}$

v t_1 je $v_y = 0 \Rightarrow \vec{r}$ má len x-ovú zložku $\vec{r}(t_1) = A\vec{i} \parallel \vec{x} \Rightarrow d = 0$

2. $m_1, m_2 \subset m_1, \vec{F}_0$

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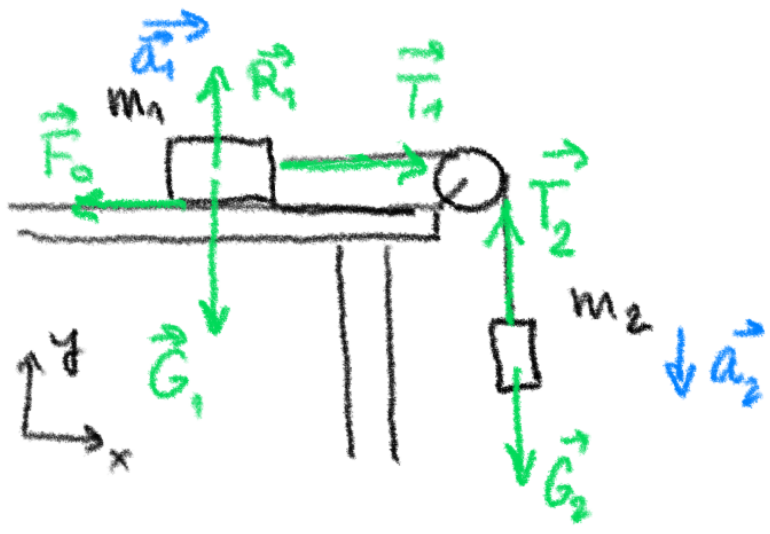
a, T



1. teleso

$m_1\vec{a}_1 = \vec{G}_1 + \vec{R}_1 + \vec{T}_1 + \vec{F}_0$

2. teleso

$m_2\vec{a}_2 = \vec{G}_2 + \vec{T}_2$

$m_1 a_1 = T_1 - F_0$

$0 = R_1 - G_1$

$-m_2 a_2 = T_2 - G_2$

$a_1 = a_2 = a$

$T_1 = T_2 = T$ → nehmotná kladka

odčítame rovnice

$$m_1 a = T - F_0$$
$$-m_2 a = T - m_2 g$$

$$\Rightarrow \quad m_1 a + m_2 a = -F_0 + m_2 g$$

$$a = \frac{m_2 g - F_0}{m_1 + m_2}$$

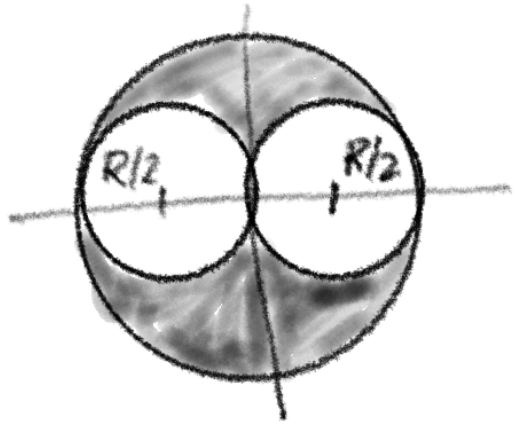
z 2. rovnice:

$$T = m_2 (g - a) = m_2 \left(g - \frac{m_2 g - F_0}{m_1 + m_2} \right)$$

$$T = m_2 \frac{m_1 g + F_0}{m_1 + m_2}$$

(3.) $R, R/2, m$

J_j



Plný disk

$$J_D = \frac{1}{2} M R^2$$

$$J = J_D - 2 J_{OT}$$

$$J = \frac{1}{2} M R^2 - \frac{3}{4} m_1 R^2$$

Steinerova veta pre 1 otvor:

$$J_{OT} = \frac{1}{2} m_1 \left(\frac{R}{2}\right)^2 + m_1 \left(\frac{R}{2}\right)^2 = \frac{3}{8} m_1 R^2$$

Plošná hustota materiálu $\sigma \quad \frac{m}{\pi R^2 - 2 S_{OT}}$

$$S_{OT} = \pi \left(\frac{R}{2}\right)^2 = \frac{\pi R^2}{4} \Rightarrow \pi R^2 - 2 \frac{\pi R^2}{4} = \frac{1}{2} \pi R^2$$

$$\sigma = \frac{m}{\frac{1}{2} \pi R^2} = \frac{2m}{\pi R^2}$$

$M = \sigma \pi R^2 = 2m$ – hmotnosť plného disku

$m_1 = \sigma \cdot \pi \left(\frac{R}{2}\right)^2 = \frac{2m}{\pi R^2} \cdot \frac{\pi R^2}{4} = \frac{1}{2} m$ – hmotnosť vyrezaného materiálu 1 otvoru

$$J = \frac{1}{2} \cdot 2 m R^2 - \frac{3}{4} \cdot \frac{1}{2} m R^2 = \frac{1}{2} m R^2 \left(2 - \frac{3}{4}\right)$$

$$J = \frac{1}{2} \cdot \frac{5}{4} m R^2 = \frac{5}{8} m R^2$$

(4.) $m, k, N; 1/10$

$f; N_1$

$$A = A_0 e^{-\gamma t}$$

$$\frac{1}{2} = e^{-\gamma N T}$$

$$\ln 1/2 = -\gamma N T$$

$$\gamma = \frac{\ln 2}{N T} = \frac{f}{N} \ln 2$$

$$\omega = \sqrt{\omega_0^2 - \beta^2} \Rightarrow \omega^2 = \omega_0^2 - \beta^2 \quad ; \quad \omega_0^2 = \frac{k}{m}$$

$$4\pi^2 f^2 = \frac{k}{m} - \frac{f^2}{N^2}\ln^2 2$$

$$4\pi^2 N^2 f^2 + f^2 (\ln 2)^2 = \frac{k}{m} N^2$$

$$f^2 = \frac{N^2 k/m}{4\pi^2 N^2 + (\ln 2)^2}$$

$$f = \sqrt{\frac{N^2 k/m}{4\pi^2 N^2 + (\ln 2)^2}}$$

$$E \sim A^2 \Rightarrow E \sim A_0^2 e^{-2\beta t}$$

$$E = E_0 e^{-2\beta t} \rightarrow \frac{1}{10} = e^{-2\beta N_1 T}$$

$$\ln \frac{1}{10} = -2\beta N_1 T$$

$$\ln 10 = 2\beta N_1 T = 2N_1 \beta / f = 2N_1 \frac{\ln 2}{N}$$

$$N_1 = \frac{N}{2} \cdot \frac{\ln 10}{\ln 2} = \frac{N}{\log 4}$$

5. $\frac{n;\ W;\ \varkappa}{\Delta T}$

$$pV^{\varkappa} = \text{konst.}$$

$$pV = nRT$$

$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} p_1 V_1^{\varkappa} \frac{1}{V^{\varkappa}} dV = p_1 V_1^{\varkappa} \int_{V_1}^{V_2} V^{-\varkappa} dV =$$

$$= p_1 V_1^{\varkappa} \left[\frac{1}{1-\varkappa} V^{1-\varkappa}\right]_{V_1}^{V_2} = \frac{p_1 V_1^{\varkappa}}{1-\varkappa}\left(V_2^{1-\varkappa} - V_1^{1-\varkappa}\right)$$

$$p_1 V_1^{\varkappa} = p_2 V_2^{\varkappa} \Rightarrow W = \frac{1}{1-\varkappa}\left(p_2 V_2^{\varkappa} V_2^{1-\varkappa} - p_1 V_1^{\varkappa} V_1^{1-\varkappa}\right) =$$

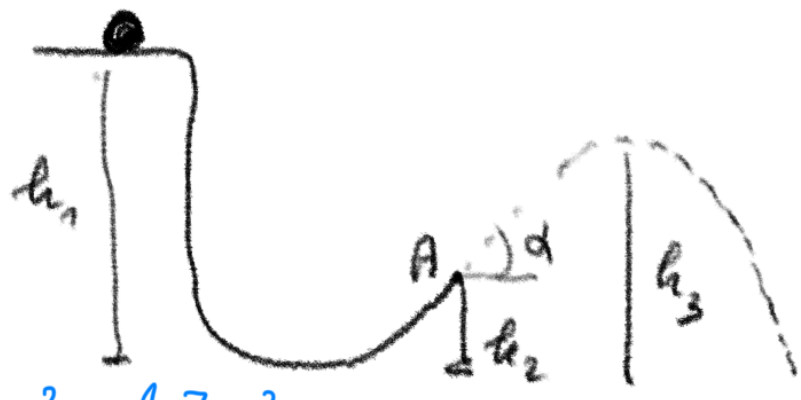
$$= \frac{1}{1-\varkappa}\left(p_2 V_2 - p_1 V_1\right) = \frac{1}{\varkappa - 1}\left(p_1 V_1 - p_2 V_2\right)$$

$$pV = nRT \Rightarrow W = \frac{1}{\varkappa - 1}\left(nRT_1 - nRT_2\right) =$$

$$= \frac{1}{\varkappa - 1} nR(T_1 - T_2) = -\frac{nR}{\varkappa - 1}\Delta T$$

$$\Delta T = -\frac{(\varkappa - 1)W}{nR}$$

6. h_1, h_2, α / $h_3, \vec{v}_3$



ZZME:

$$mgh_1 = mgh_2 + \frac{1}{2}mv_2^2 + \frac{1}{2}Jw_2^2$$

$$mgh_2 + \frac{1}{2}mv_2^2 = mgh_3 + \frac{1}{2}mv_3^2$$ (tu sa rotácia guličky nemení)

$J = \frac{2}{5}mR^2$, $w_2 = \frac{v_2}{R} \Rightarrow$

$$\Rightarrow E_{k2} = \frac{1}{2}mv_2^2 + \frac{1}{2}Jw_2^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}\frac{2}{5}mR^2\frac{v_2^2}{R^2} = \frac{7}{10}mv_2^2$$

$$mgh_1 = mgh_2 + \frac{7}{10}mv_2^2 \Rightarrow v_2^2 = \frac{10}{7}g(h_1 - h_2)$$

Po opustení dráhy je to šikmý vrh. V najvyššom bode je $v_y = 0 \Rightarrow v_3 = v_2\cos\alpha$

$$mgh_2 + \frac{1}{2}mv_2^2 = mgh_3 + \frac{1}{2}mv_3^2$$

$$mgh_2 + \frac{1}{2}mv_2^2 = mgh_3 + \frac{1}{2}m \cdot v_2^2\cos^2\alpha$$

$$h_3 = h_2 + \frac{1}{2g}v_2^2(1 - \cos^2\alpha) = h_2 + \frac{1}{2}\frac{v^2}{g}\sin^2\alpha$$

$$h_3 = h_2 + \frac{1}{2}\cdot\frac{10}{7}(h_1 - h_2)\sin^2\alpha$$

$$h_3 = h_2 + \frac{5}{7}(h_1 - h_2)\sin^2\alpha$$

$$\vec{v}_3 = (v_2\cos\alpha)\vec{i} = \sqrt{\frac{10}{7}g(h_1 - h_2)}\cos\alpha\,\vec{i}$$