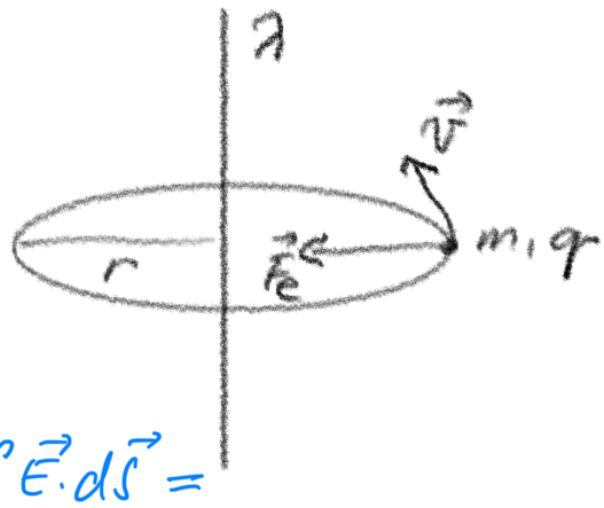
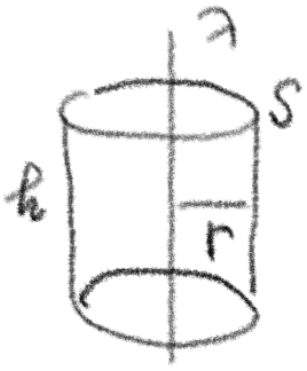
F2 2025 ZS S3 Riešenie

1. $\lambda > 0,\ q < 0,\ m$

$E;\ v$





$$\oint_S \vec{E}\cdot d\vec{S} = \frac{Q}{\varepsilon_0}$$

$$\oint_S \vec{E}\cdot d\vec{S} = \int_{\text{podstavy}} \vec{E}\cdot d\vec{S} + \int_{\text{plášť}} \vec{E}\cdot d\vec{S} =$$

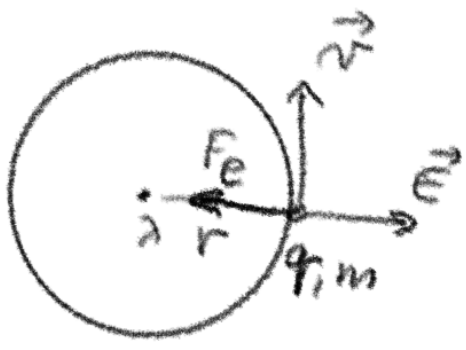
podstavy: $\vec{E} \perp d\vec{S} \Rightarrow \vec{E}\cdot d\vec{S} = 0$

plášť: $\vec{E} \parallel d\vec{S} \Rightarrow \vec{E}\cdot d\vec{S} = E\,dS$

$$\oint_S \vec{E}\cdot d\vec{S} = \int_{\text{plášť}} \vec{E}\cdot d\vec{S} = \int_{\text{plášť}} E\,dS = E\int_{\text{plášť}} dS = E\,2\pi r\cdot h$$

$$Q = \lambda h \rightarrow 2\pi r\, h\, E = \frac{\lambda h}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$



$$\vec{F_e} = q\vec{E} = \vec{F_d}$$

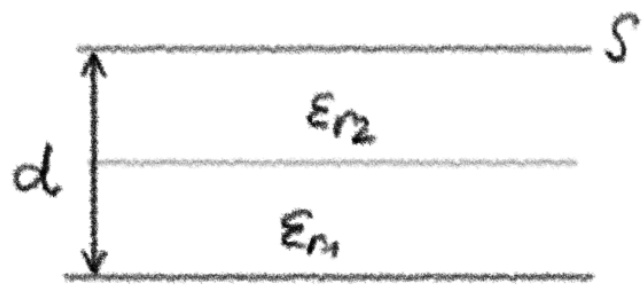
$$qE = m\frac{v^2}{r}$$

$$q\,\frac{\lambda}{2\pi\varepsilon_0 r} = m\frac{v^2}{r}$$

$$v^2 = \frac{\lambda q}{2\pi\varepsilon_0 m} \Rightarrow v = \sqrt{\frac{\lambda q}{2\pi\varepsilon_0 m}}$$

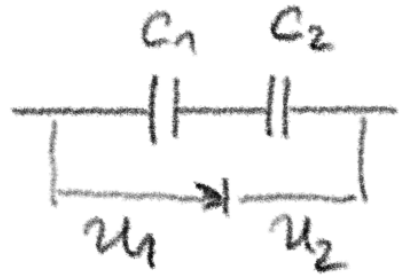
2. $S, d, \varepsilon_{r1}, \varepsilon_{r2}$

$C;\ \sigma_v$



$$C = \frac{Q}{U}$$

$$U = \int_0^d E\,dy$$



$$U = \int_0^{d/2} E_1\,dy + \int_{d/2}^{d} E_2\,dy = U_1 + U_2$$

Zapojenie do série: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

$$C_1 = \varepsilon_0 \varepsilon_{r1} \frac{2S}{d} \qquad C_2 = \varepsilon_0 \varepsilon_{r2} \frac{2S}{d}$$

$$\frac{1}{C} = \frac{d}{2\varepsilon_0 \varepsilon_{r1} S} + \frac{d}{2\varepsilon_0 \varepsilon_{r2} S} = \frac{d}{2\varepsilon_0 S}\left(\frac{1}{\varepsilon_{r1}} + \frac{1}{\varepsilon_{r2}}\right)$$

$$C = \varepsilon_0 \frac{\varepsilon_{r1}\,\varepsilon_{r2}}{\varepsilon_{r1} + \varepsilon_{r2}} \frac{2S}{d}$$

$$\sigma_V = P_{n1} - P_{n2}$$

Vektory $\vec{E}, \vec{D}, \vec{P}$ majú iba normálové zložky vzhľadom na rozhranie. Ak sú platne nabité nábojom $\pm Q$, plošné hustoty náboja:

$$\sigma_+ = \frac{Q}{S} \qquad \sigma_- = -\frac{Q}{S}$$

normálová zložka $D_n = D_{n2} = \sigma = \frac{Q}{S}$

normálová zložka $E_{n1} = \frac{D_{n1}}{\varepsilon_0 \varepsilon_{r1}}$; $E_{n2} = \frac{D_{n2}}{\varepsilon_0 \varepsilon_{r2}}$

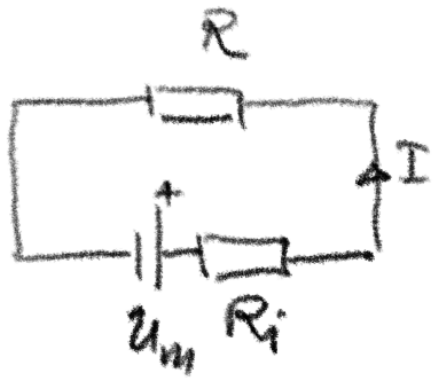
$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \Rightarrow \vec{P} = \vec{D} - \varepsilon_0 \vec{E}$$

$$P_{n1} = D_{n1} - \varepsilon_0 E_1 = \sigma - \frac{\sigma}{\varepsilon_{r1}} \quad ; \quad P_{n2} = \sigma - \frac{\sigma}{\varepsilon_{r2}}$$

$$\sigma_V = P_{n1} - P_{n2} = \sigma\left(\frac{1}{\varepsilon_{r2}} - \frac{1}{\varepsilon_{r1}}\right) = \sigma \frac{\varepsilon_{r1} - \varepsilon_{r2}}{\varepsilon_{r1}\,\varepsilon_{r2}}$$

3. R_1, I_1, R_2, I_2

I_3) P



$U_m = I_1 (R_1 + R_i)$

$U_m = I_2 (R_2 + R_i)$

$U_m = I_3 R_i \qquad (R_3 = 0\,\Omega)$

$I_3 = \frac{U_m}{R_i}$ $\qquad I_1 (R_1 + R_i) = I_2 (R_2 + R_i)$

$$R_i = \frac{I_2 R_2 - I_1 R_1}{I_1 - I_2}$$

$$U_m = I_1 R_1 + I_1 R_i = I_1 R_1 + I_1 \frac{I_2 R_2 - I_1 R_1}{I_1 - I_2} = \frac{I_1^2 R_1 - I_1 I_2 R_1 + I_1 I_2 R_2 - I_1^2 R_1}{I_1 - I_2} =$$

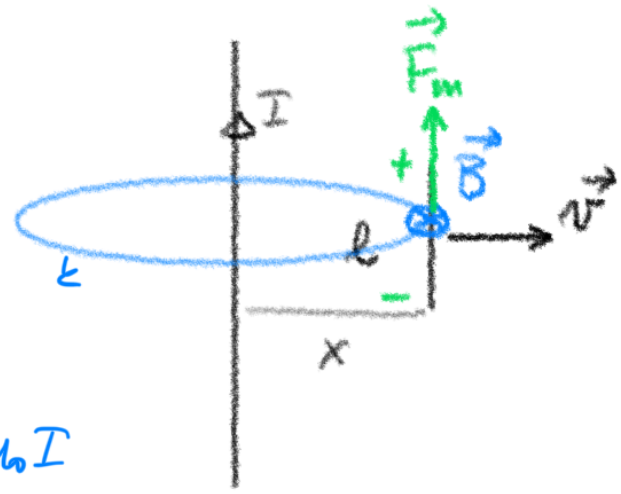
$$= \frac{I_1 I_2}{I_1 - I_2} (R_2 - R_1)$$

$$I_3 = \frac{I_1 I_2 (R_2 - R_1)}{I_1 - I_2} \cdot \frac{I_1 - I_2}{I_2 R_2 - I_1 R_1} = \frac{I_1 I_2}{I_2 R_2 - I_1 R_1} (R_2 - R_1)$$

$$P = U_m I_3 = R_i \cdot I_3^2 = \frac{(I_1 I_2)^2 (R_2 - R_1)^2}{(I_1 - I_2)(I_2 R_2 - I_1 R_1)}$$

4. x, I, ℓ, v

U_i



$\oint_k \vec{B} \cdot d\vec{\ell} = \mu_0 I$

$\oint_k \vec{B} \cdot d\vec{\ell} = B 2\pi x = \mu_0 I$

$B = \frac{\mu_0 I}{2\pi x}$

V drôte pôsobí na nosiče náboja magnetická sila

$\vec{F}_m = q \vec{v} \times \vec{B}$

indukovaný náboj vytvoří elektrické pole $\vec{E_i}$:

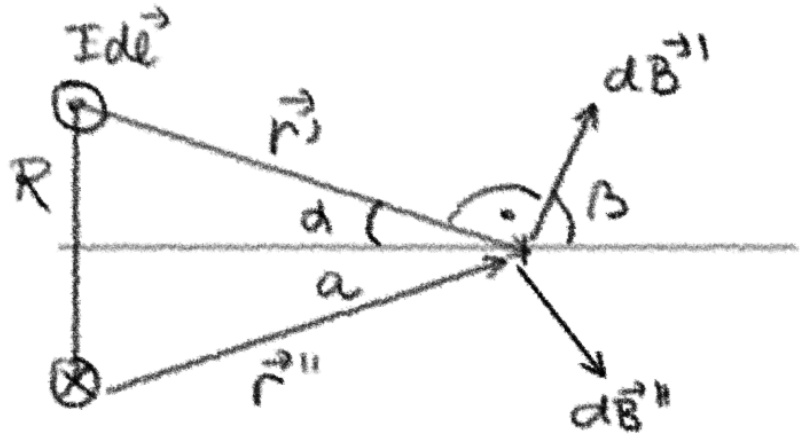
$$q(\vec{v}\times\vec{B}) + q\vec{E_i} = \vec{0}$$

$$\vec{v}\perp\vec{B} \Rightarrow \quad E_i = vB$$

$$U_i = \int_0^e E_i\,dl = \int_0^e vB\,dl = vBl$$

(5.) R, a, I

B_j



$$d\vec{B'} = \frac{\mu_0}{4\pi} I \frac{d\vec{l}\times\vec{r'}}{r'^3} \quad ; \quad dB' = \frac{\mu_0}{4\pi}\frac{Idl}{r'^2}$$

$$d\vec{B} = d\vec{B}' + d\vec{B}'', \; dB = 2dB'\cos\beta = \frac{\mu_0}{2\pi}\frac{I}{r'^2}\cos\beta\,dl$$

$$B = \frac{\mu_0 I}{2\pi r'^2}\cos\beta\int_k dl = \frac{\mu_0 I}{2\pi r'^2}\cos\beta\int_0^{\pi} R\,d\varphi = \frac{\mu_0 I R}{2r'^2}\cos\beta$$

$$r'^2 = R^2 + a^2$$

$$\cos\beta = \cos\left(\pi - \tfrac{\pi}{2} - \alpha\right) = \cos\left(\tfrac{\pi}{2} - \alpha\right) = \sin\alpha$$

$$\sin\alpha = \frac{R}{r'} = \frac{R}{\sqrt{R^2+a^2}} = \cos\beta$$

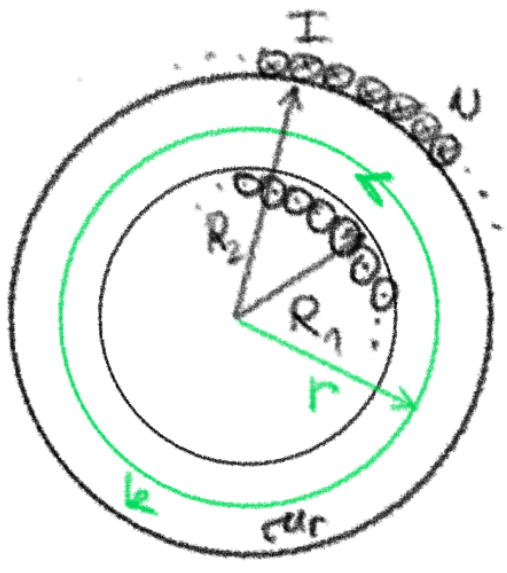
$$B = \frac{\mu_0 I R}{2}\frac{R}{(R^2+a^2)^{3/2}} = \frac{\mu_0 I}{2R}\frac{1}{(1+a^2/R^2)^{3/2}}$$

(6.) N, R_1, R_2, a, μ_r

L

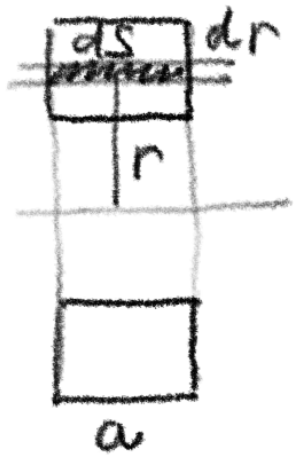
$$L = \frac{\Phi}{I}$$

$$\Phi_0 = \int \vec{B}\cdot d\vec{S}$$



$$\oint_L \vec{B}\cdot d\vec{l} = \mu_0 N I \qquad \vec{B} \parallel d\vec{l} \Rightarrow \vec{B}\cdot d\vec{l} = B\,dl$$

$$\oint_L B\,dl = B\oint_L dl = B\,2\pi r = \mu_0 N I \Rightarrow B = \frac{\mu_0 N I}{2\pi r}$$



$$\Phi_0 = \int_S \vec{B}\cdot d\vec{S} = \int_S B\,dS \qquad \vec{B} \parallel d\vec{S}$$

$$dS = a\,dr$$

$$\Phi_0 = \int_{R_1}^{R_2} \frac{\mu_0 N I}{2\pi r}\,a\,dr = \frac{\mu_0 N I}{2\pi}\,a \ln R_2/R_1$$

$$\Phi = N\Phi_0 = \frac{\mu_0 N^2 I a}{2\pi} \ln R_2/R_1$$

$$L = \frac{\Phi}{I} = \frac{\mu_0}{2\pi} N^2 I a \ln R_2/R_1$$