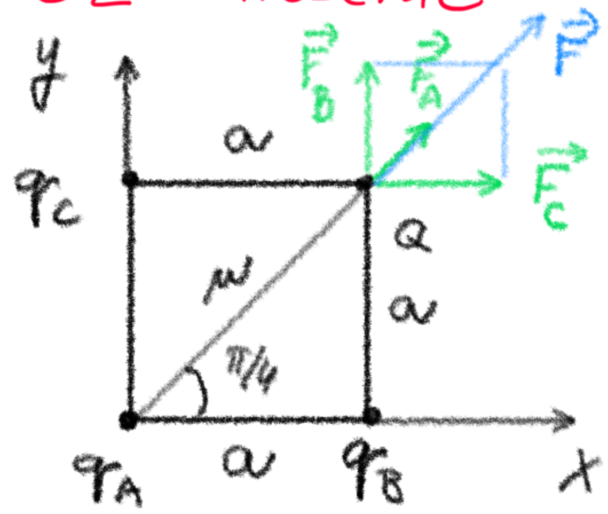
F2 2025 LS S2 Riešenie



1. $q_i\ Q$

$\vec{F}_j$

$$\vec{F} = \vec{F}_A + \vec{F}_B + \vec{F}_C$$

$$\vec{F}_A = \frac{qQ}{4\pi\varepsilon_0}\frac{1}{r^2}\left(\vec{i}\cos\frac{\pi}{4} + \vec{j}\sin\frac{\pi}{4}\right) = \frac{qQ}{4\pi\varepsilon_0}\frac{1}{\sqrt{2}\,r^2}(\vec{i}+\vec{j})$$

$$\vec{F}_B = \frac{qQ}{4\pi\varepsilon_0}\frac{1}{a^2}\vec{j}$$

$$\vec{F}_C = \frac{qQ}{4\pi\varepsilon_0}\frac{1}{a^2}\vec{i}$$

$$r^2 = 2a^2$$

$$\vec{F} = \frac{qQ}{4\pi\varepsilon_0 a^2}\left[\left(\frac{1}{2\sqrt{2}}+1\right)\vec{i} + \left(\frac{1}{2\sqrt{2}}+1\right)\vec{j}\right]$$

$$\vec{F} = \frac{qQ}{4\pi\varepsilon_0 a^2}\left(1+\frac{\sqrt{2}}{4}\right)(\vec{i}+\vec{j})$$

$$F = \frac{qQ}{4\pi\varepsilon_0 a^2}\left(1+\frac{\sqrt{2}}{4}\right)\sqrt{1+1} = \frac{qQ}{4\pi\varepsilon_0 a^2}\left(\sqrt{2}+\frac{1}{2}\right)$$

2. $\varphi = -ax^2 - by^2$; $a, b > 0$

$\vec{r}_A = x_1\vec{i} + y_1\vec{j}$

$\vec{r}_B = x_2\vec{i} + y_2\vec{j}$

$q = e$

$\vec{E}$, W

$$\vec{E} = -\operatorname{grad}\varphi$$

$$E_x = -\frac{\partial\varphi}{\partial x} = -\frac{\partial}{\partial x}(-ax^2-by^2) = 2ax$$

$$E_y = -\frac{\partial\varphi}{\partial y} = -\frac{\partial}{\partial y}(-ax^2-by^2) = 2by$$

$$E_z = -\frac{\partial\varphi}{\partial z} = 0$$

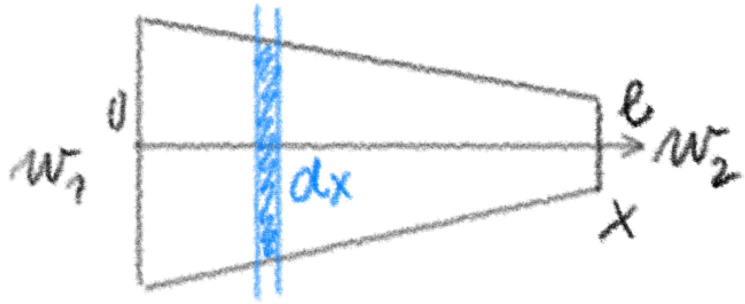
$$\vec{E} = 2ax\,\vec{i} + 2by\,\vec{j}$$

$$W = q(\varphi_A - \varphi_B) = q(-ax_1^2 - by_1^2 + ax_2^2 + by_2^2)$$

$$W = q\left[a(x_2^2 - x_1^2) + b(y_2^2 - y_1^2)\right]$$

3. $h,\ w_1,\ w_2 < w_1,\ l,\ \rho$

$R,$



$$dR = \rho \frac{dx}{h\,w(x)} \qquad w(x) = kx + q$$

$$dR = \rho \frac{dx}{h(kx+q)}$$

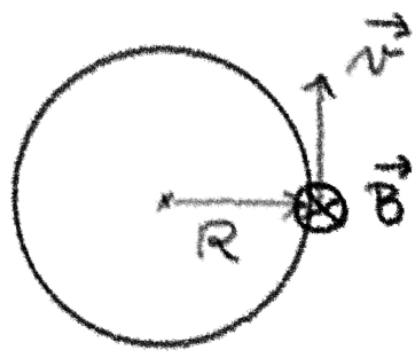
$$R = \frac{\rho}{h} \int_0^l \frac{dx}{(kx+q)} = \left| \begin{matrix} w = kx+q \\ dw = k\,dx \end{matrix} \right| = \frac{\rho}{hk} \int_{w_1}^{w_2} \frac{dw}{w} =$$

$$= \frac{\rho}{h\,k} \ln w_2/w_1$$

$$k = \frac{w_2 - w_1}{l} \qquad R = \frac{\rho\, l}{h\,(w_2 - w_1)} \ln w_2/w_1$$

4. $q,\ m,\ \vec{B},\ R$

$E_k;\ u$



kružnica $\Rightarrow$ $\vec{v} \perp \vec{B}$

$\vec{F} = q\vec{v} \times \vec{B} = \vec{F_d}$ $\rightarrow$ dostrediva sila

$$F = qvB = m\frac{v^2}{R} \qquad E_k = \frac{1}{2}mv^2$$

$$v = \frac{qBR}{m} \qquad E_k = \frac{1}{2} m \left(\frac{qBR}{m}\right)^2$$

$$E_k = \frac{q^2B^2R^2}{2m}$$

Pri urýchľovaní z pokoja:

$$\overset{=0}{E_{k1}} + E_{p1} = E_{k2} + E_{p2}$$

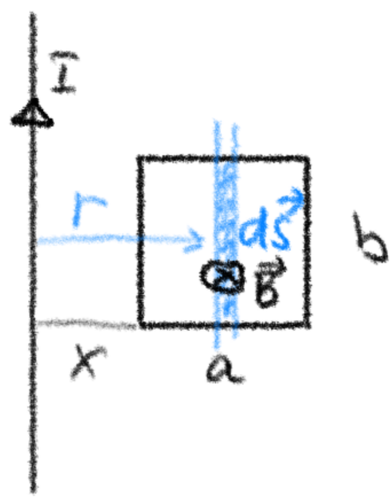
$$E_{k2} = E_k = E_{p1} - E_{p2} = qU$$

$$E_k = qU \Rightarrow \quad U = \frac{E_k}{q}$$

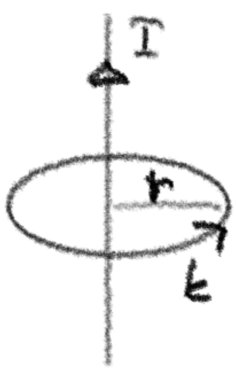
$$U = \frac{qB^2R^2}{2m}$$

5. $a, b, x, I = I_0\left(1 - e^{-t/\tau}\right)$

$M, \; U_i$



Pole dlhého vodiča



$$\oint_l \vec{B}\cdot d\vec{l} = \mu_0 I$$

$$\vec{B} \parallel d\vec{l}$$

$$\oint_l \vec{B}\cdot d\vec{l} = \oint_l B\,dl = B\oint_l dl = B\,2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

vzájomná indukčnosť $\quad M = \dfrac{\Phi_2}{I_1}$

$$\Phi_2 = \int_S \vec{B}\cdot d\vec{S}, \quad \vec{B} \parallel d\vec{S} \Rightarrow \vec{B}\cdot d\vec{S} = B\,dS$$

$$dS = b\,dr \quad \Rightarrow \quad B\,dS = B\,b\,dr$$

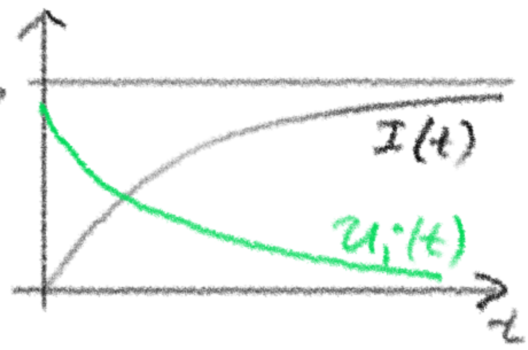
$$\Phi_2 = \int_x^{x+a} B\,b\,dr = \int_x^{x+a} \frac{\mu_0 I}{2\pi r}\,b\,dr = \frac{\mu_0 I}{2\pi}\,b\,\ln\left(1 + a/x\right)$$

$$M = \frac{\Phi_2}{I} = \frac{\mu_0}{2\pi}\,b\,\ln\left(1 + a/x\right)$$

$$u_i = -\frac{d\Phi_2}{dt} = -M\frac{dI}{dt} = -\frac{\mu_0}{2\pi} b \ln(1+a/x)\frac{d}{dt} I_0(1-e^{-t/\tau})$$

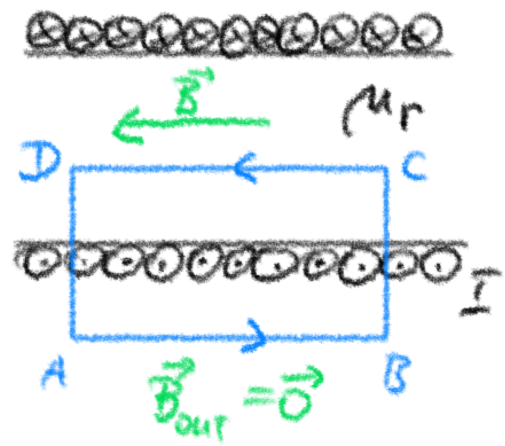
$$\frac{d}{dt} I_0(1-e^{-t/\tau}) = -\frac{I_0}{\tau} e^{-t/\tau}$$

$$u_i = \frac{\mu_0 I_0}{2\pi\tau} b \ln(1+a/x) e^{-t/\tau}$$



6. μ_r, n, l, R

$B;\ w_m;\ W_m$



$$\oint_{ABCD} \vec{H}\cdot d\vec{l} = I_c = NI$$

$$N = |CD|\, n$$

na úseku AB je $\vec{H} = \vec{0}$ (mimo cievky)

na úsekoch BC a DA je $\vec{H} \perp d\vec{l} \Rightarrow \vec{H}\cdot d\vec{l} = 0$

na úseku CD je $\vec{H} \uparrow\uparrow d\vec{l} \Rightarrow \vec{H}\cdot d\vec{l} = H\,dl$

$$\oint_{ABCD} \vec{H}\cdot d\vec{l} = \int_{CD} H\,dl = H|CD|$$ (homogénne pole)

$$H|CD| = |CD|\,nI \quad\Rightarrow\quad H = nI$$

$$B = \mu_0\mu_r H \quad\Rightarrow\quad B = \mu_0\mu_r nI$$

$$w_m = \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{1}{2}BH = \frac{1}{2}B\frac{B}{\mu_0\mu_r} = \frac{1}{2}\frac{B^2}{\mu_0\mu_r}$$

$$w_m = \frac{1}{2}\mu_0\mu_r n^2 I^2$$

$$W = \int_V w_m\,dV = w_m V$$ (homogénne pole)

$$W = w_B \cdot \pi R^2 \cdot l = \frac{1}{2}\mu_0\mu_r n^2 I^2 \pi R^2 l = \frac{\pi}{2}\mu_0\mu_r R^2 l n^2 I^2$$