F2 2025 LS S1 RIEŠENIE

1. $x_1, q_1 > 0, \; x_2, q_2 < 0$

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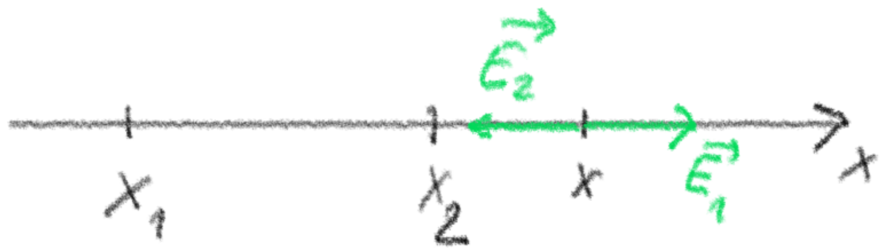
x;



$x \notin \langle x_1; x_2 \rangle$

$$\vec{E}_1 + \vec{E}_2 = \vec{0} \qquad E_{1x} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{(x-x_1)^2}$$

$$E_{2x} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(x-x_2)^2}$$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1}{(x-x_1)^2} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(x-x_2^2)} = 0$$

$$q_1(x-x_2)^2 + q_2(x-x_1)^2 = 0$$

$$q_1(x^2 - 2xx_2 + x_2^2) + q_2(x^2 - 2xx_1 + x_1^2) = 0$$

$$x^2(q_1+q_2) - 2x(q_1x_2 + q_2x_1) + q_1x_2^2 + q_2x_1^2 = 0$$

$$D = 4(q_1x_2 + q_2x_1)^2 - 4(q_1+q_2)(q_1x_2^2 + q_2x_1^2) =$$

$$= \cancel{4q_1^2x_2^2} + \cancel{4q_2^2x_1^2} + 8q_1q_2x_1x_2 - \cancel{4q_1^2x_2^2} - \cancel{4q_2^2x_1^2} -$$

$$- 4q_1q_2x_1^2 - 4q_1q_2x_2^2 =$$

$$= -4q_1q_2(x_1^2 + x_2^2 - 2x_1x_2) = -4q_1q_2(x_2-x_1)^2 \geq 0$$

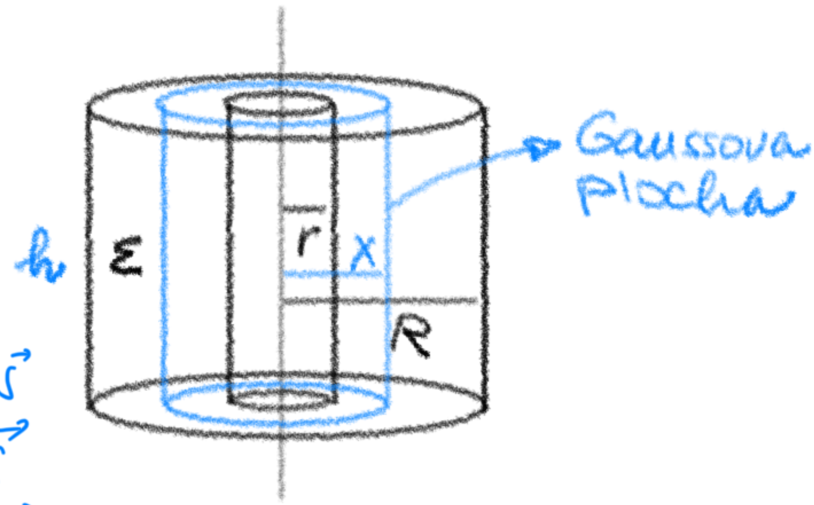
lebo $q_1q_2 < 0$

$$x_{1,2} = \frac{2(q_1x_2 + q_2x_1) \pm 2|x_2-x_1|\sqrt{-q_1q_2}}{2(q_1+q_2)}$$

$$x_{1,2} = \frac{q_1x_2 + q_2x_1 \pm |x_2-x_1|\sqrt{-q_1q_2}}{q_1+q_2}$$

2. $\quad r, R, \varepsilon$

$\overline{C}$



$$\oint_S \vec{D}\cdot d\vec{S} = Q$$

$$\oint_S \vec{D}\cdot d\vec{S} = \underbrace{\int_{\text{plášť}} \vec{D}\cdot d\vec{S}}_{\vec{D}\parallel d\vec{S}} + \underbrace{\int_{\text{podstavy}} \vec{D}\cdot d\vec{S}}_{\vec{D}\perp d\vec{S}} = 0$$

$$\oint_S \vec{D}\cdot d\vec{S} = \int_{\text{plášť}} \vec{D}\cdot d\vec{S} = \int_{\text{plášť}} D\, dS = D\int_{\text{plášť}} dS = D\, 2\pi x h$$

$$D\, 2\pi x h = Q \quad \Rightarrow \quad D = \frac{Q}{2\pi x h}$$

$$D = \varepsilon E \quad \Rightarrow \quad E = \frac{D}{\varepsilon} = \frac{Q}{2\pi\varepsilon x h}$$

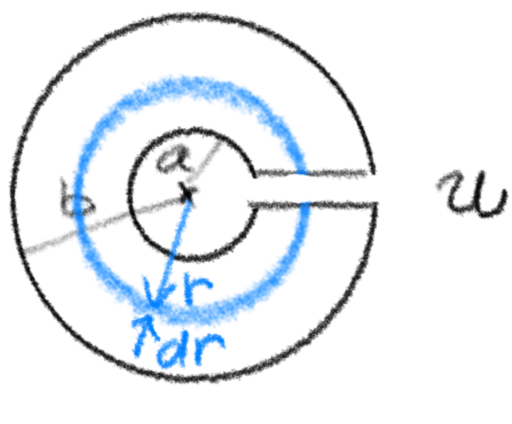
$$U = \int_r^R E\, dx = \int_r^R \frac{Q\, dx}{2\pi\varepsilon x h} = \frac{Q}{2\pi\varepsilon h}\left[\ln x\right]_r^R =$$

$$= \frac{Q}{2\pi\varepsilon h}\ln R/r$$

$$C = \frac{Q}{U} = \frac{2\pi\varepsilon h}{\ln R/r} \qquad c = \frac{C}{h} = \frac{2\pi\varepsilon}{\ln R/r}$$

3. $\quad a, b, h, \varrho, U$

$\overline{I,\ P}$



$$I = \frac{U}{R} = UG \qquad P = \frac{U^2}{R} = GU^2$$

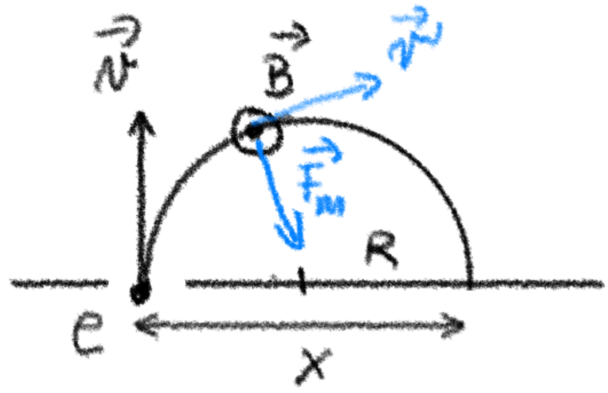
$$dG = \frac{1}{\varrho}\,\frac{h\, dr}{2\pi r}$$

$$G = \frac{h}{2\pi\varrho}\int_a^b \frac{dr}{r} = \frac{h}{2\pi\varrho}\ln b/a$$

$$I = \frac{hU}{2\pi\varrho}\ln b/a \qquad P = \frac{hU^2}{2\pi\varrho}\ln b/a$$

4. $U, e, \vec{B}, x$

m_p



$$F_m = evB$$
$$F_d = m_p \frac{v^2}{R}$$

$$\Delta E_k + \Delta E_p = 0$$
$$\frac{1}{2} m_p v^2 - eU = 0$$

$$R = \frac{x}{2}$$

$$v^2 = \frac{2eU}{m_p}$$

$$F_m = F_d$$

$$v = \sqrt{\frac{2eU}{m_p}}$$

$$e \cdot \sqrt{\frac{2eU}{m_p}}\, B = \frac{2eU}{x/2}$$

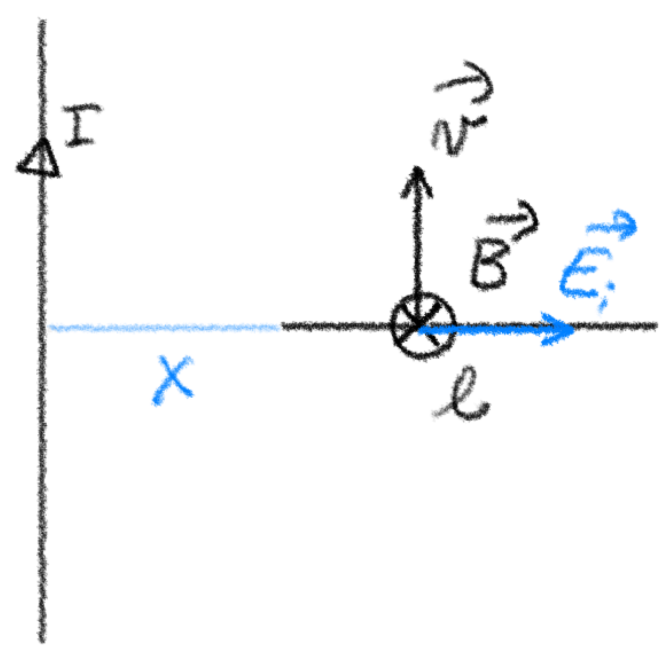
$$B\sqrt{\frac{2eU}{m_p}} = \frac{4U}{x}$$

$$B^2 \frac{2eU}{m_p} = \frac{16U^2}{x^2}$$

$$m_p = \frac{eB^2x^2}{8U}$$

5. $I, x, l, \vec{v}$

U_i



$$B = \frac{\mu_0 I}{2\pi r}$$

$$\vec{E}_i = \vec{v} \times \vec{B}$$

$$U_i = \int_l \vec{E} \cdot d\vec{r} = \int_l (\vec{v} \times \vec{B}) \cdot d\vec{r} =$$

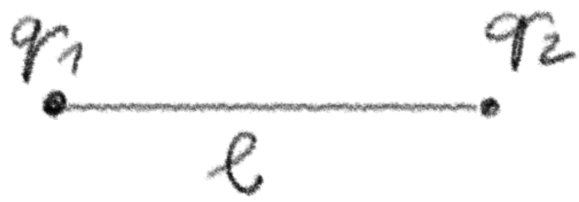
$$= \int_x^{x+l} v B \, dr = v \int_x^{x+l} \frac{\mu_0 I}{2\pi r} dr =$$

$$= \frac{\mu_0}{2\pi} I v \int_x^{x+l} \frac{dr}{r} = \frac{\mu_0 I v}{2\pi} \left[\ln \right]_x^{x+l}$$

$$U_i = \frac{\mu_0 I v}{2\pi} \ln (1 + l/x)$$

6. $\underline{q_1, q_2, m, l}$

v_1, v_2



$$E_{p1} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{l} \qquad E_{k1} = 0$$

$$E_{p2} = 0 \qquad E_{k2} = \frac{1}{2} m (v_1^2 + v_2^2)$$

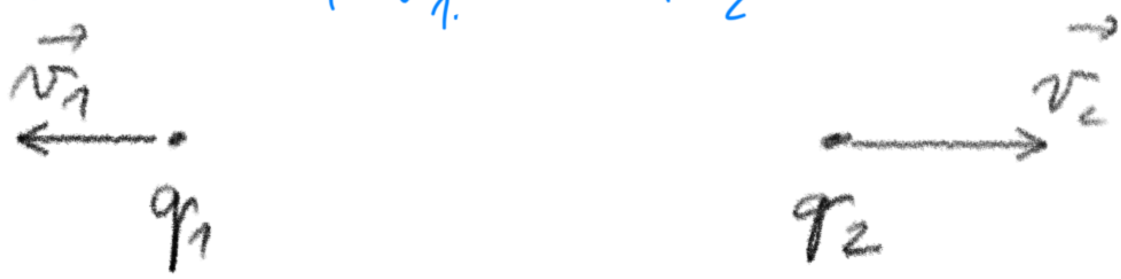
Zákon zachovania energie:

$$E_{p1} + E_{k1} = E_{p2} + E_{k2}$$

$$\frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{l} = \frac{1}{2} m (v_1^2 + v_2^2)$$

Zákon zachovania hybnosti:

$$0 = m\vec{v}_1 + m\vec{v}_2$$



$$\vec{v}_1 = -v_1 \vec{i} \qquad \vec{v}_2 = v_2 \vec{i}$$

$$0 = m(-v_1 \vec{i}) + m v_2 \vec{i}$$

$$m(v_2 - v_1) = 0 \Rightarrow v_1 = v_2 = v$$

$$\frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{\ell} = \frac{1}{2} m 2v^2 = mv^2$$

$$v = \sqrt{\frac{q_1 q_2}{4\pi\varepsilon_0 m \ell}}$$