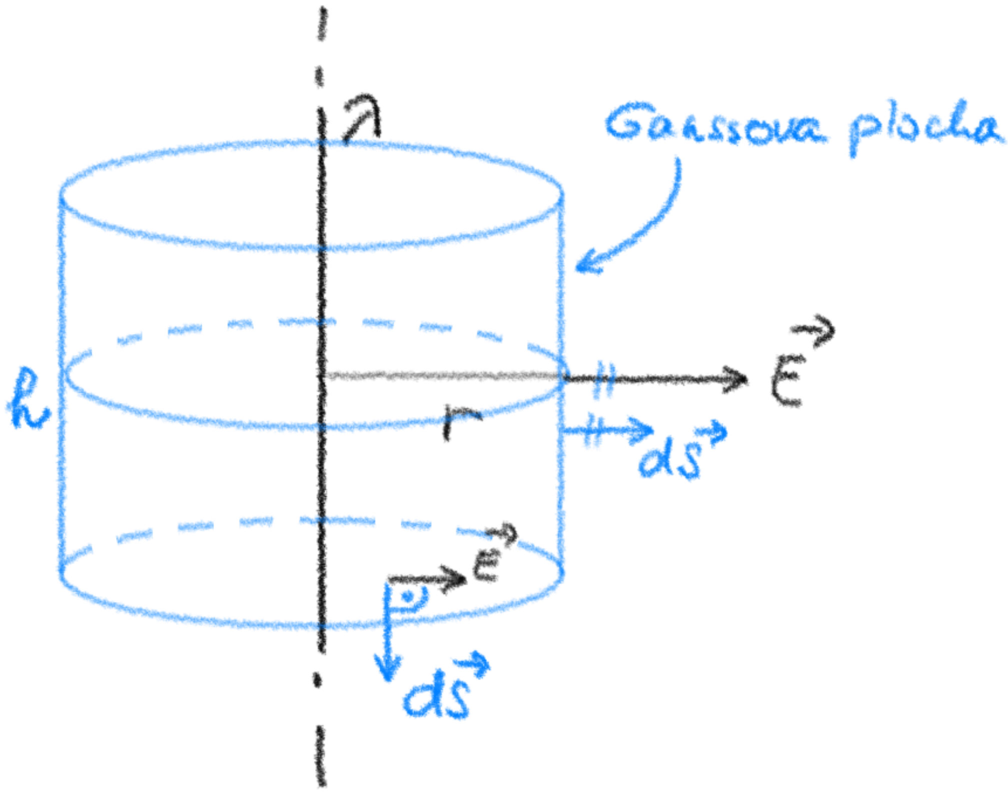
F2 2025 ZS PÍSOMKA 1 – RIEŠENIE

① $\dfrac{\lambda, r}{E}$



$$\oint_S \vec{E}\cdot d\vec{S} = \frac{Q}{\varepsilon_0}$$

$$\oint_S \vec{E}\cdot d\vec{S} = \int_{\text{plášť}} \vec{E}\cdot d\vec{S} + \int_{\text{podstavy}} \vec{E}\cdot d\vec{S} = \int_{\text{plášť}} E\, dS$$

plášť: $\vec{E} \uparrow\uparrow d\vec{S} \Rightarrow \vec{E}\cdot d\vec{S} = E\, dS$

podstavy: $\vec{E} \perp d\vec{S} \Rightarrow \vec{E}\cdot d\vec{S} = 0$

$$\int_{\text{plášť}} E\, dS = E \int_{\text{plášť}} dS = E\, 2\pi r h = \frac{Q}{\varepsilon_0}$$

$Q \rightarrow$ náboj uzavretý v Gaussovej ploche

$$Q = \lambda h$$

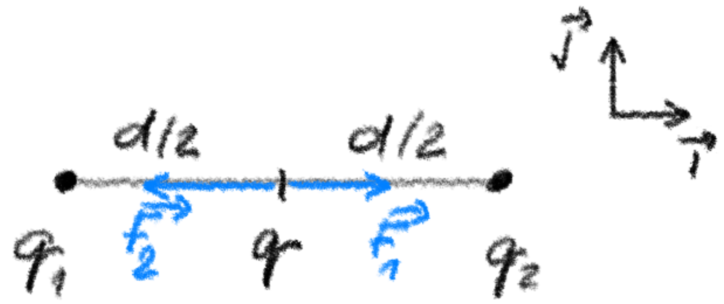
$$\oint_S \vec{E}\cdot d\vec{S} = E\, 2\pi r h = \frac{Q}{\varepsilon_0} = \frac{1}{\varepsilon_0}\lambda h$$

$$E\, 2\pi r h = \frac{\lambda h}{\varepsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi \varepsilon_0 r}}$$

2. $q, q_1, q_2, q_1 q_2 > 0, d$

F;



$$\vec{F} = \vec{F_1} + \vec{F_2} \qquad \vec{F_1} = F_1 \vec{i} \; ; \; \vec{F_2} = -F_2 \vec{i}$$

$$\vec{F_1} = \frac{1}{4\pi\varepsilon_0} \frac{|q q_1|}{d^2/4} \vec{i} = \frac{1}{\pi\varepsilon_0} \frac{|q q_1|}{d^2}$$

$$\vec{F_2} = -\frac{1}{4\pi\varepsilon_0} \frac{|q q_2|}{d^2/4} \vec{i} = -\frac{1}{\pi\varepsilon_0} \frac{|q q_2|}{d^2}$$

$$\vec{F} = \left(\frac{1}{\pi\varepsilon_0} \frac{|q q_1|}{d^2} - \frac{1}{\pi\varepsilon_0} \frac{|q q_2|}{d^2} \right) \vec{i} = \frac{|q|}{\pi\varepsilon_0 d^2} \left(|q_1| - |q_2| \right) \vec{i}$$

$$F = \frac{|q|}{\pi\varepsilon_0 d^2} |q_1 - q_2|$$

3. $\varphi(x) = E x, \; q$

$\vec{r_1} = x_1 \vec{i} + y_1 \vec{j}$

$\vec{r_2} = x_2 \vec{i} + y_2 \vec{j}$

$\vec{E}, \; U, \; W$

$$\vec{E} = -\operatorname{grad} \varphi$$

$$E_x = -\frac{\partial \varphi}{\partial x} = -E$$

$$E_y = \frac{\partial \varphi}{\partial y} = 0 \; ; \quad E_z = \frac{\partial \varphi}{\partial z} = 0$$

$$\vec{E} = -E \vec{i}$$

$$U_{21} = \varphi(\vec{r_2}) - \varphi(\vec{r_1}) = \varphi(x_2) - \varphi(x_1) = E x_2 - E x_1$$

$$U_{21} = E(x_2 - x_1)$$

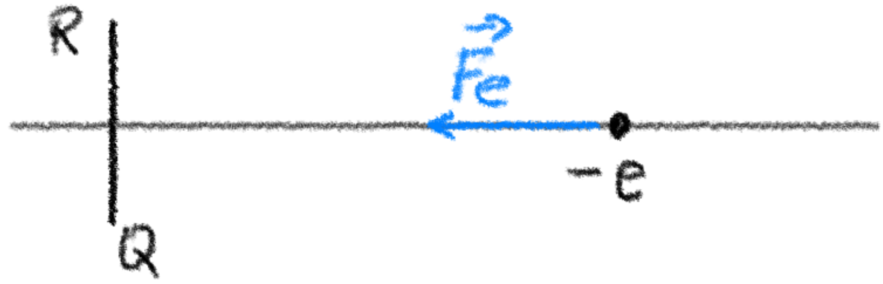
$$W_{21} = q\,U_{21} = qE(x_2 - x_1)$$

Kto má rád integrály: $W = -\int_{r_1}^{r_2} \vec{F}\cdot d\vec{l} =$

$$= -\int_{x_1}^{x_2} q(-E)\,dx = qE(x_2 - x_1)$$

4. $R,\ Q>0,\ q_e = -e,\ m_e$

$v;$



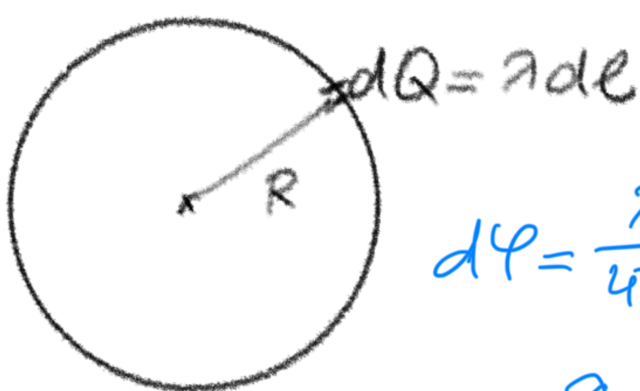
v nekonečne: $E_{k1} = 0\,J;\ E_{p1} = 0\,J$

v strede kružnice: $E_{k2} = \frac{1}{2} m_e v^2;\ E_p = -e\varphi$

ZZE: $0 = \frac{1}{2} m_e v^2 - e\varphi$

$$\Rightarrow \quad v^2 = \frac{2e\varphi}{m_e}$$

$\varphi \rightarrow$ potenciál el. poľa v strede kružnice:



$$d\varphi = \frac{\lambda dl}{4\pi\varepsilon_0 R}$$

$$\varphi = \frac{\lambda}{4\pi\varepsilon_0 R}\int_k dl = \frac{\lambda}{4\pi\varepsilon_0 R}2\pi R$$

$$\lambda = \frac{Q}{2\pi R}$$

$$\varphi = \frac{Q/2\pi R}{4\pi\varepsilon_0 R} 2\pi R = \underline{\underline{\frac{Q}{4\pi\varepsilon_0 R}}}$$

$$v^2 = \frac{2e}{m_e} \cdot \frac{Q}{4\pi\varepsilon_0 R} = \frac{eQ}{2\pi\varepsilon_0 m_e R}$$

$$v = \sqrt{\frac{eQ}{2\pi\varepsilon_0 m_e R}}$$