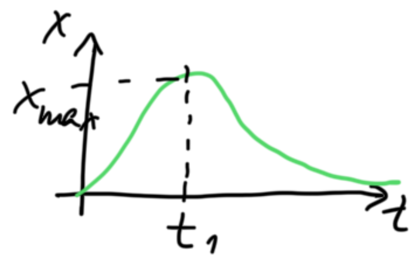
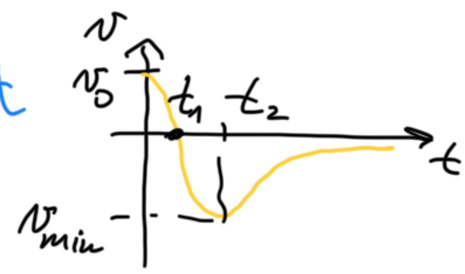
F1 LS 2025 Skúška 2 riešenie

1. $x = v_0 t e^{-\gamma t}$

$v(t), a(t), t_1, t_2, x(0), v(0), a(0)$



$$v = \frac{dx}{dt} = v_0 (1 - \gamma t) e^{-\gamma t}$$



$$a = \frac{dv}{dt} = v_0 \gamma (\gamma t - 2) e^{-\gamma t}$$

$$x_{max} \Leftrightarrow \frac{dx}{dt} = 0 \Rightarrow v = 0 : v_0 (1 - \gamma t_1) e^{-\gamma t_1} = 0$$

$$1 - \gamma t_1 = 0 \quad \Rightarrow \quad t_1 = 1/\gamma$$

$$v_{min} \Leftrightarrow \frac{dv}{dt} = 0 \Rightarrow a = 0 : v_0 \gamma (\gamma t_2 - 2) e^{-\gamma t_2} = 0$$

$$\gamma t_2 - 2 = 0 \quad \Rightarrow \quad t_2 = 2/\gamma = 2 t_1$$

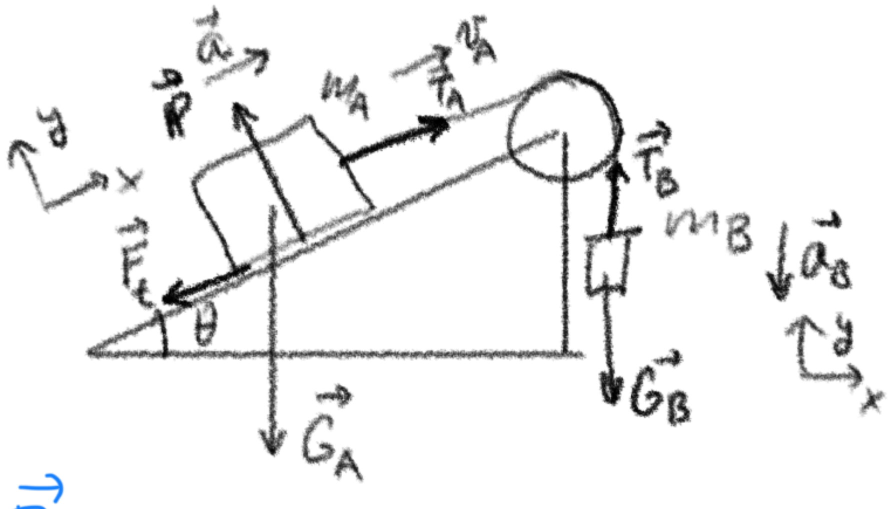
$$x(0) = v_0 0 \, e^{-\gamma \cdot 0} = 0$$

$$v(0) = v_0 (1 - \gamma \cdot 0) e^{-\gamma \cdot 0} = v_0$$

$$a(0) = v_0 \gamma (\gamma \cdot 0 - 2) e^{-\gamma \cdot 0} = -2 v_0 \gamma$$

② m_A, m_B, μ, θ

a;



Teleso A:

$$m_A\vec{a}_A = \vec{G}_A + \vec{T}_A + \vec{F}_t + \vec{R}$$

Teleso B:

$$m_B\vec{a}_B = \vec{G}_B + \vec{T}_B$$

A) $m_A a_A = -m_A g\sin\theta + T_A - F_t$

$\quad 0 = -m_A g\cos\theta + R$

B) $-m_B a_B = -m_B g + T_B$

$a_A = a_B = a \qquad T_A = T_B = T \qquad F_t = \mu R$

$$m_A a = -m_A g\sin\theta + T - \mu R$$
$$0 = -m_A g\cos\theta + R$$
$$-m_B a = -m_B g + T$$

$$R = m_A g\cos\theta$$
$$T = m_B(g-a)$$

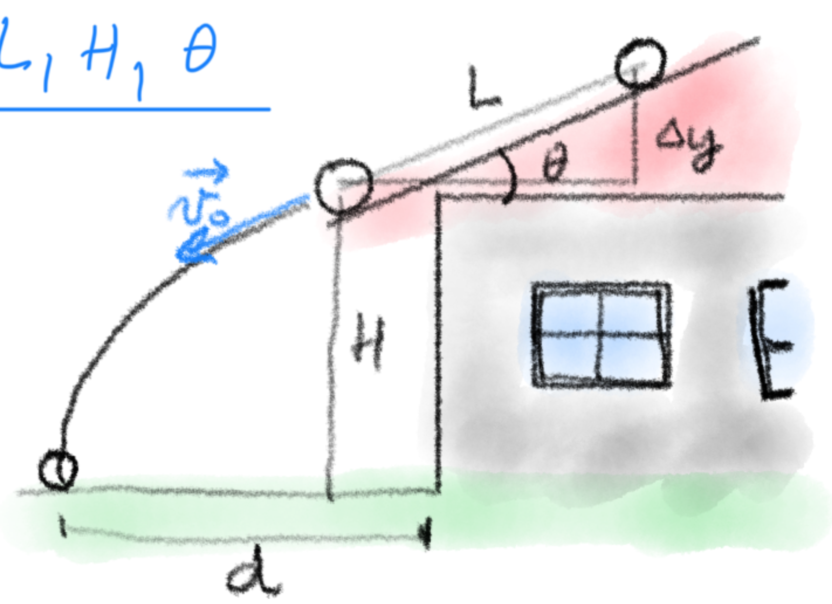
$$m_A a = -m_A g\sin\theta + m_B(g-a) + \mu m_A g\cos\theta$$

$$m_A a + m_B a = m_A g(\mu\cos\theta - \sin\theta) + m_B g$$

$$a = \frac{m_A(\mu\cos\theta - \sin\theta) + m_B}{m_A + m_B}\, g$$

3. $m, R, J = \frac{2}{5}mR^2, L, H, \theta$

$\omega; d$



ZZE: $E_{p1} + E_{k1} = E_{p2} + E_{k2}$

$E_{k1} = 0$

$\Rightarrow E_{k2} = E_{p1} - E_{p2}$

$$E_{k2} = mg\Delta y = mgL\sin\theta$$

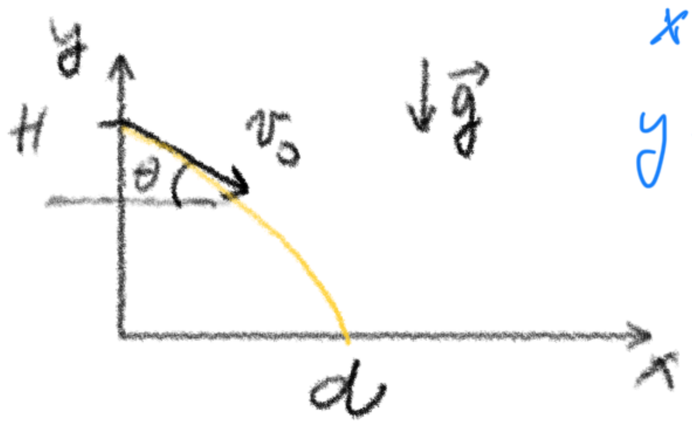
$$E_{k2} = \frac{1}{2}mv^2 + \frac{1}{2}J\omega^2 \qquad v = R\omega$$

$$\frac{1}{2}\cancel{m}R^2\omega^2 + \frac{1}{2}\frac{2}{5}\cancel{m}R^2\omega^2 = \cancel{m}gL\sin\theta$$

$$\frac{1}{2}R^2\omega^2\left(1 + \frac{2}{5}\right) = gL\sin\theta$$

$$\frac{7}{10}R^2\omega^2 = gL\sin\theta \Rightarrow \omega = \sqrt{\frac{10gL\sin\theta}{7R^2}}$$

- Šikmý vrh:



$$x = (v_0\cos\theta)t$$

$$y = H - (v_0\sin\theta)t - \frac{1}{2}gt^2$$

pričom $v_0 = R\omega$

dopad: $d = (v_0\cos\theta)t$

$$0 = H - (v_0\sin\theta)t - \frac{1}{2}gt^2$$

$$D = v_0^2 \sin^2\theta + 4H\tfrac{1}{2}g = v_0^2\sin^2\theta + 2Hg$$

$$t_{1,2} = \frac{v_0 \sin\theta \pm \sqrt{v_0^2\sin^2\theta + 2Hg}}{-g}$$

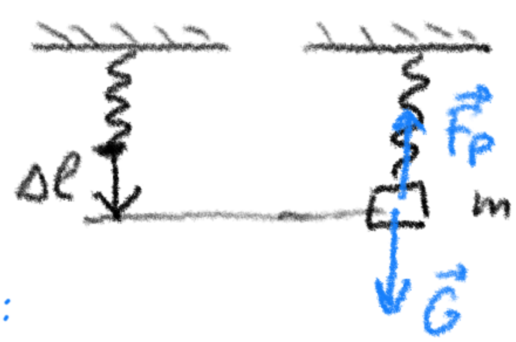
$$t > 0 \Rightarrow t = \frac{v_0 \sin\theta}{g}\left(\sqrt{1+\frac{2Hg}{v_0^2\sin^2\theta}} - 1\right)$$

$$d = \frac{v_0^2 \sin\theta\cos\theta}{g}\left(\sqrt{1+\frac{2Hg}{v_0^2\sin^2\theta}} - 1\right)$$

$$v_0^2 = R^2\omega^2 = \frac{10}{7} gL\sin\theta$$

$$d = \frac{10}{7} L\sin^2\theta\cos\theta\left(\sqrt{1+\frac{7H}{5L\sin^3\theta}} - 1\right)$$

4. $\dfrac{\Delta l,\ m}{f,\ E,\ x_m,\ v_m,\ a_m}$



Závažie na pružine v pokoji:

$$\vec{G} + \vec{F}_p = \vec{0}$$

$$-mg + k\Delta l = 0$$

$$k = \frac{mg}{\Delta l} \qquad \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg}{m\Delta l}}$$

$$\omega = \sqrt{\frac{g}{\Delta l}} \Rightarrow f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta l}}$$

$$y_m = \Delta l \qquad E = \frac{1}{2}ky_m^2 = \frac{1}{2}\frac{mg}{\Delta l}\Delta l^2 = \frac{1}{2}mg\Delta l$$

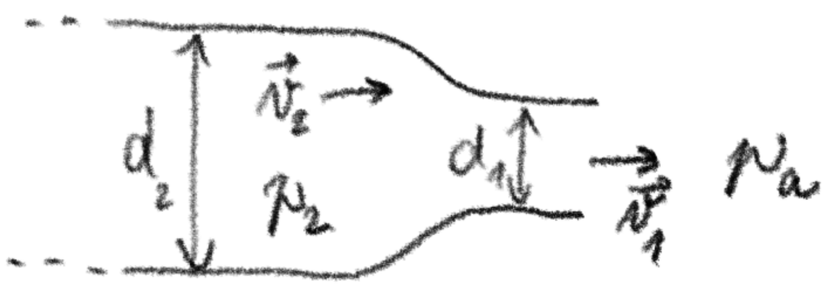
$$E = \frac{1}{2}mv_m^2 \Rightarrow v_m^2 = \frac{2E}{m} = \frac{2\frac{1}{2}mg\Delta l}{m} = g\Delta l$$

$$v_m = \sqrt{g\Delta l}$$

$$a_m = \omega^2 y_m = \frac{g}{\Delta l}\Delta l = g = a_m$$

5. v_1, d_1, d_2, p_a, t

V, v_2, p_2



$$V = q_V \cdot t \qquad q_V = S v \qquad S = \frac{1}{4}\pi d^2$$

$$V = S_1 v_1 \cdot t = \frac{\pi}{4} d_1^2 \cdot v_1 \cdot t$$

Rovnica spojitosti $S_1 v_1 = S_2 v_2$

$$\frac{\pi}{4} d_1^2 v_1 = \frac{\pi}{4} d_2^2 v_2 \Rightarrow v_2 = v_1 \frac{d_1^2}{d_2^2}$$

Bernouilliho rovnica

$$\frac{1}{2}\rho v_2^2 + p_2 = \frac{1}{2}\rho v_1^2 + p_a$$

$$p_2 = p_a + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$

$$p_2 = p_a + \frac{1}{2}\rho\left(v_1^2 - v_1^2 \frac{d_1^4}{d_2^4}\right)$$

$$p_2 = p_a + \frac{1}{2}\rho v_1^2 \left(1 - d_1^4/d_2^4\right)$$

6. $n, V_1, T_1, \quad V_2 = \frac{2}{3}V_1, \quad p = konst.$

T_2, W'

$$pV = nRT \qquad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$T_2 = \frac{V_2}{V_1} T_1 = \frac{2}{3} T_1$$

Práca plynu $W = \int_{V_1}^{V_2} p\, dV = p(V_2 - V_1)$

Práca vonk. sily $W' = -W = p(V_1 - V_2)$

$$W' = p\left(V_1 - \frac{2}{3}V_1\right) = \frac{1}{3} p V_1$$

$$W' = \frac{1}{3} nRT_1$$