F1 LS 2025 Skúška 1 Riešenie

1. $v = k t^{1/2}$

r

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a_t, ε, a_d, a

$$a_t = \frac{dv}{dt} = \frac{d}{dt} k t^{1/2} = k \frac{1}{2} t^{-1/2} = \frac{k}{2\sqrt{t}}$$

$$\varepsilon = \frac{a_t}{r} = \frac{k}{2r\sqrt{t}}$$

$$a_d = \frac{v^2}{r} = k^2 \frac{t}{r}$$

$$a = \sqrt{a_t^2 + a_d^2} = \sqrt{\frac{k^2}{4t} + k^4 \frac{t^2}{r^2}} = k\sqrt{\frac{1}{4t} + \frac{k^2}{r^2} t^2}$$

2. m_2, ℓ, m_1, α

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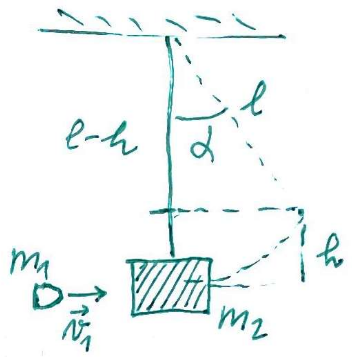
v_1;



nepružná zrážka:

$$m_1 \vec{v}_1 = (m_1 + m_2) \vec{v}_2$$

$$v_2 = \frac{m_1}{m_1 + m_2} v_1$$

zákon zach. mech. energie pri stúpaní debničky:

$$\frac{1}{2}(m_1+m_2) v_2^2 + E_{p1} = E_{p2}$$

$$\frac{1}{2}(m_1+m_2) v_2^2 = E_{p2} - E_{p1} = (m_1+m_2) g h$$

$$\frac{1}{2} \cancel{(m_1+m_2)} \frac{m_1^2}{(m_1+m_2)^2} v_1^2 = \cancel{(m_1+m_2)} g h$$

$$v_1^2 = 2\left(\frac{m_1+m_2}{m_1}\right)^2 g h$$

$$\ell \cos\alpha = \ell - h$$

$$h = \ell(1 - \cos\alpha)$$

$$v_1 = \frac{m_1+m_2}{m_1} \sqrt{2 g \ell (1 - \cos\alpha)}$$

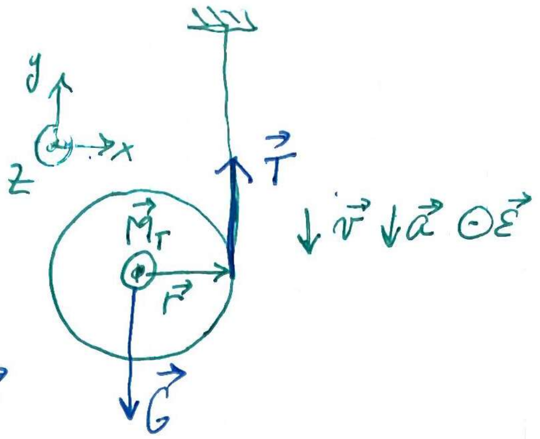
3. $J = \frac{1}{2} m R^2$

$\overline{a, T}$

$|\vec{r}| = R$

$\vec{T} + \vec{G} = m\vec{a}$

$\vec{M}_T + \vec{M}_G = J\vec{\varepsilon}$ $\qquad \vec{M}_G = \vec{0}$



$y: \; T - mg = -ma$

$z: \; TR = J\varepsilon$ $\qquad \varepsilon = \frac{a}{R}$

$$\begin{aligned} mg - T &= ma \\ TR &= \tfrac{1}{2} mR^2 \cdot \tfrac{a}{R} \end{aligned} \Rightarrow \begin{aligned} mg - T &= ma \\ T &= \tfrac{1}{2} ma \end{aligned}$$

$\cancel{m}g - \frac{1}{2}\cancel{m}a = \cancel{m}a$

$a(1 + \frac{1}{2}) = g \Rightarrow a = \frac{2}{3} g$

$T = \frac{1}{3} mg$

4. W, m, T

$\overline{k; A; v_m}$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

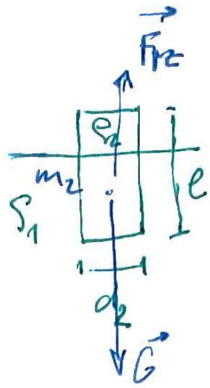
$k = \frac{4\pi^2}{T^2} m$

$W = \frac{1}{2} kA^2 \Rightarrow A = \sqrt{\frac{2W}{k}} = \frac{T}{2\pi}\sqrt{\frac{2W}{m}}$

$W = \frac{1}{2} m v_m^2 \Rightarrow v_m = \sqrt{\frac{2W}{m}}$

(5.) $d_1, h_1, d_2 < d_1, \rho_1, \rho_2, l$

h_2, m_1



$\vec{G} + \vec{F}_{vz} = \vec{0}$

$-m_2 g + F_{vz} = 0$

$F_{vz} = m_2 g$

$m_2 = \rho_2 \cdot V_2$

$V_2 = \pi \frac{d_2^2}{4} \cdot l$

↳ objem valčeka

$F_{vz} = V_p \cdot \rho_1 \cdot g = V_2 \rho_2 g$

V_p - objem ponorenej časti valčeka

$$V_p = V_2 \frac{\rho_2}{\rho_1} = \frac{\pi}{4} d_2^2 \cdot l \frac{\rho_2}{\rho_1}$$

Voda vo valci stúpne o Δh, pričom

$$V_p = \Delta h \cdot S_1 = \Delta h \cdot \frac{\pi}{4} d_1^2$$

$$\Delta h = \frac{4 V_p}{\pi d_1^2} = \frac{4 \pi d_2^2 l \rho_2}{\pi d_1^2 4 \rho_1} = \frac{d_2^2}{d_1^2} \cdot \frac{\rho_2}{\rho_1} \cdot l$$

$$h_2 - h_1 = \Delta h \Rightarrow h_2 = \Delta h + h_1 = \frac{d_2^2}{d_1^2} \cdot \frac{\rho_2}{\rho_1} \cdot l + h_1$$

Z valca treba odliať V_p vody, čo zodpovedá hmotnosti $m_1 = \rho_1 V_p$

$$m_1 = \rho_1 \cdot \frac{\pi}{4} d_2^2 \cdot l \frac{\rho_2}{\rho_1} = \frac{\pi}{4} d_2^2 \cdot l \rho_2 = m_2$$

Z valca treba odliať toľko vody, koľko váži valček, lebo $V_p \rho_1 g = m_2 g \Rightarrow V_p \rho_1 = m_1 = m_2$.

⑥ $n, T_1, V_1, V_2 = \frac{1}{2}V_1, \varkappa$

$W; T_2$

adiab. zákon:

$p_1 V_1^{\varkappa} = p_2 V_2^{\varkappa}$

$p V^{\varkappa} = p_1 V_1^{\varkappa}$

$\Rightarrow p = p_1 V_1^{\varkappa} V^{-\varkappa}$

$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} p_1 V_1^{\varkappa} V^{-\varkappa}\,dV =$$

$$= p_1 V_1^{\varkappa} \int_{V_1}^{V_2} V^{-\varkappa}\,dV = p_1 V_1 \frac{1}{1-\varkappa}\left[V^{1-\varkappa}\right]_{V_1}^{V_2} =$$

$$= p_1 V_1^{\varkappa} \frac{1}{1-\varkappa}\left(V_2^{1-\varkappa} - V_1^{1-\varkappa}\right) = \frac{1}{1-\varkappa}\left(p_1 V_1^{\varkappa} V_1^{1-\varkappa} - p_1 V_1^{\varkappa} V_2^{1-\varkappa}\right) =$$

$$= \frac{1}{1-\varkappa}\left(p_1 V_1 - p_2 V_2\right) = \frac{1}{\varkappa - 1}\left(p_2 V_2 - p_1 V_1\right)$$

Stavová rovnica: $pV = nRT$

$$W = \frac{1}{\varkappa-1}\left(nRT_2 - nRT_1\right) = \frac{nR\,(T_2 - T_1)}{\varkappa - 1}$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \Rightarrow T_2 = \frac{p_2 V_2}{p_1 V_1} T_1$$

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\varkappa} \quad \rightarrow \quad T_2 = \left(\frac{V_1}{V_2}\right)^{\varkappa} \cdot \frac{V_2}{V_1} T_1 = \left(\frac{V_1}{V_2}\right)^{\varkappa-1} T_1 = 2^{\varkappa-1}\, T_1$$

$$\underline{\underline{T_2 = 2^{\varkappa-1}\, T_1}}$$

$$\underline{\underline{W = \frac{2^{\varkappa-1} - 1}{\varkappa - 1}\, nRT_1}}$$