F1 LS 2025 P3 Riešenie

1. $x = At$
$y = Bt - Ct^3$

$\vec{r}, \vec{a}, r(0)$

$$\vec{r} = \frac{d\vec{r}}{dt}$$

$$r_x = \frac{dx}{dt} = \frac{d}{dt}(At) = A$$

$$r_y = \frac{dy}{dt} = \frac{d}{dt}(Bt - Ct^3) = B - 3Ct^2$$

$$\vec{a} = \frac{d\vec{r}}{dt} \qquad a_x = \frac{dr_x}{dt} = \frac{dA}{dt} = 0$$

$$a_y = \frac{dr_y}{dt} = \frac{d}{dt}(B - 3Ct^2) = -6Ct$$

$$\vec{r} = A\vec{i} + (B - 3Ct^2)\vec{j}$$

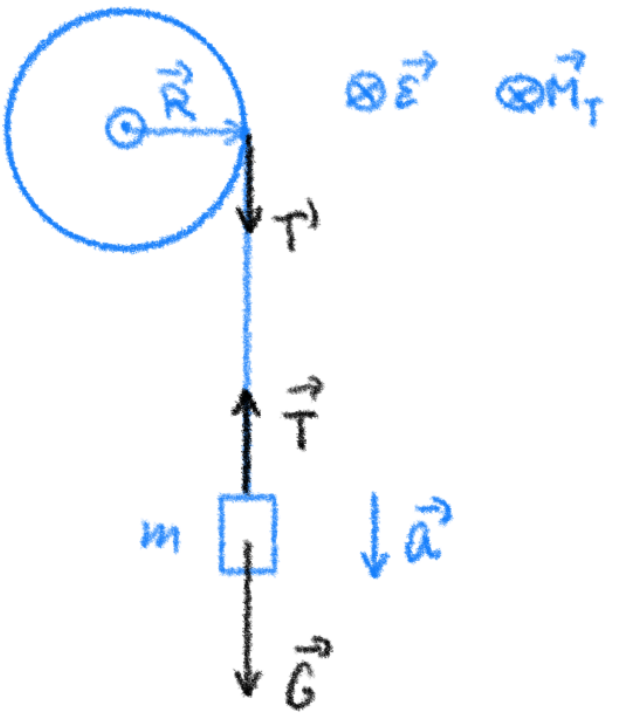
$$\vec{a} = -6Ct\vec{j}$$

$$r = \sqrt{r_x^2 + r_y^2} = \sqrt{A^2 + (B - 3Ct^2)^2}$$

$$r(0) = \sqrt{A^2 + B^2}$$

2. R, m, a

J, T



$$m\vec{a} = \vec{G} + \vec{T}$$
$$J\vec{\varepsilon} = \vec{M}_T$$

$$ma = mg - T \qquad T = T'$$
$$\varepsilon = \frac{a}{R}$$

$$J\varepsilon = T'R$$

$$ma = mg - T$$
$$J\frac{a}{R} = TR$$

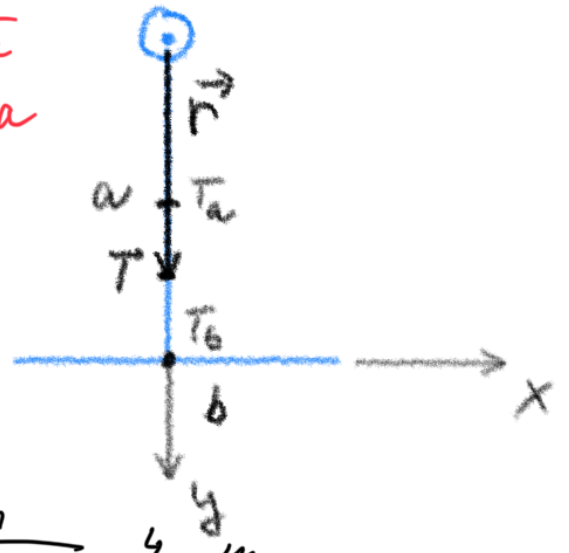
$$T = m(g - a)$$

$$Ja = TR^2 \Rightarrow J = \frac{TR^2}{a} = mR^2\frac{g-a}{a}$$

$$J = \left(\frac{g}{a} - 1\right)mR^2$$

3. $a = \ell$
$b = \frac{3}{4}\ell$

Při tomto zadání mi trochu uletela ruka.



$J_T = \frac{1}{12} m\ell^2$

$f_j \quad f = \frac{1}{2\pi}\sqrt{\frac{mgr}{J}}$

$m = m_a + m_b \qquad \lambda = \frac{m}{a+b} = \frac{m}{\ell + \frac{3}{4}\ell} = \frac{4}{7}\frac{m}{\ell}$

$m_a = a\lambda = \ell \cdot \frac{4}{7}\frac{m}{\ell} = \frac{4}{7}m$

$m_b = b\lambda = \frac{3}{4}\ell\,\frac{4}{7}\frac{m}{\ell} = \frac{3}{7}m$

$\vec{r} = \frac{m_a \vec{r_a} + m_b \vec{r_b}}{m}$, $\vec{r_a} = \frac{\ell}{2}\cdot\vec{j}$; $\vec{r_b} = \ell\vec{j}$

$\vec{r} = \frac{\frac{4}{7}m\,\frac{1}{2}\ell + \frac{3}{7}m\cdot\ell}{m}\vec{j} = \frac{5}{7}\ell\vec{j} \Rightarrow r = \frac{5}{7}\ell$

$J = J_a + J_b$; $J_a = \frac{1}{12}m_a\ell^2 + m_a\left(\frac{\ell}{2}\right)^2 = \frac{1}{3}m_a\ell^2 = \frac{4}{7}\frac{1}{3}m\ell^2$

$J_b = \frac{1}{12}m_b b^2 + m_b\ell^2 = \frac{1}{12}m_b\left(\frac{3}{4}\ell\right)^2 + m_b\ell^2 = \frac{67}{64}m_b\ell^2 = \frac{3}{7}\frac{67}{64}m\ell^2$

$J = \frac{1}{7}\left(\frac{4}{3} + \frac{3\cdot 67}{64}\right)m\ell^2 = \frac{1}{7}\frac{859}{192}m\ell^2$

$f = \frac{1}{2\pi}\sqrt{\frac{mg\cdot\frac{5}{7}\ell}{\frac{1}{7}\frac{859}{192}m\ell^2}} = \frac{1}{2\pi}\sqrt{\frac{960}{859}\frac{g}{\ell}}$

4. $d_1,\ d_2 < d_1,\ F,\ S$

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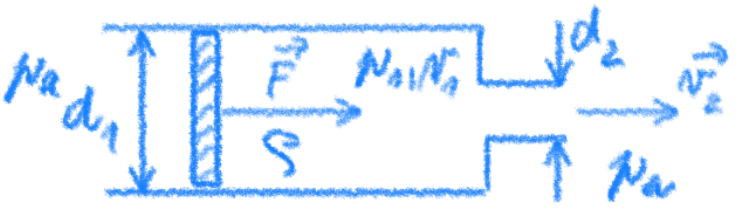
v_2 ;



$S = \frac{1}{4}\pi d^2$

$v_1 = \frac{F}{S_1}$

$S_1 v_1 = S_2 v_2$

$p_1 + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2$

$$v_1 = \frac{S_2}{S_1} v_2 = \frac{d_2^2}{d_1^2} v_2$$

$$p_1 + \frac{1}{2}\rho\left(\frac{d_2^2}{d_1^2} v_2\right)^2 = \frac{1}{2}\rho v_2^2$$

$$p_1 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho\left(\frac{d_2}{d_1}\right)^4 v_2^2 = \frac{1}{2}\rho v_2^2\left(1 - \frac{d_2^4}{d_1^4}\right)$$

$$v_2 = \sqrt{\frac{2p_1}{\rho}\frac{d_1^4}{d_1^4 - d_2^4}} = \sqrt{\frac{8F}{\rho\pi d_1^2}\frac{d_1^4}{d_1^4 - d_2^4}}$$

$$v_2 = 2d_1\sqrt{\frac{2F}{\pi\rho(d_1^4 - d_2^4)}}$$