Fyzika 1 LS 2024 Skúška 2 – Riešenie

1.

$$v(t) = \sqrt{2kt}$$

$$x(0) = x_0$$

$$\overline{x(t),\ a(t)}$$

$$x(t) = \int v(t)\,dt$$

$$x = \int \sqrt{2kt}\,dt =$$

$$= \sqrt{2k}\int t^{1/2}dt = \sqrt{2k}\,\frac{3}{2}t^{3/2} + C = 3\sqrt{\frac{1}{2}kt^3} + C$$

$$x(0) = 3\sqrt{\frac{1}{2}k\cdot 0^3} + C = x_0 \Rightarrow C = x_0$$

$$x(t) = 3\sqrt{\frac{1}{2}kt^3} + x_0$$

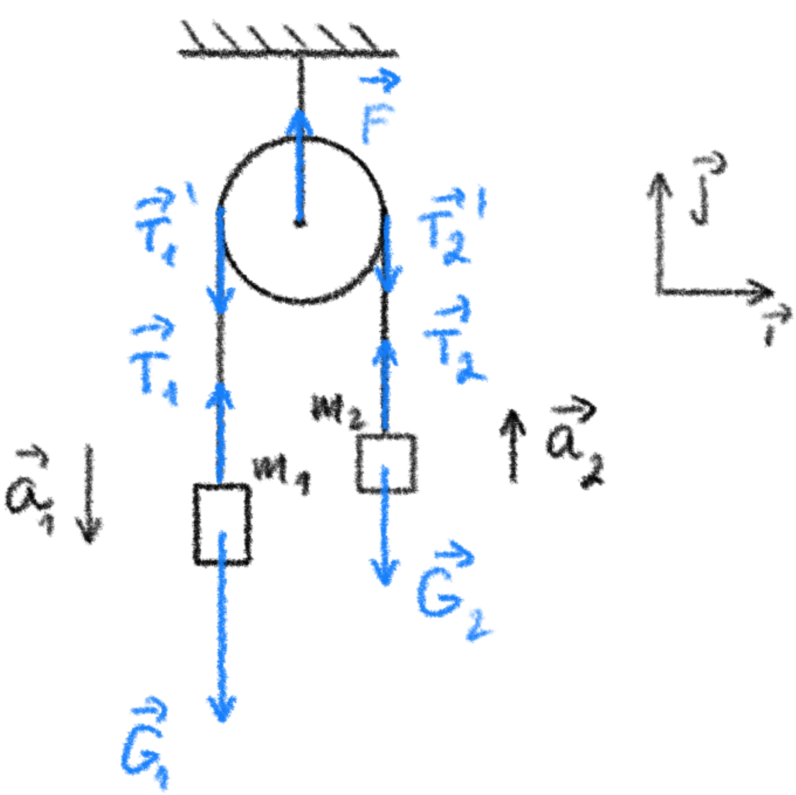
$$a = \frac{dv}{dt} = \frac{d}{dt}\sqrt{2kt} = \sqrt{2k}\,\frac{d}{dt}t^{1/2} = \sqrt{2k}\,\frac{1}{2}t^{-1/2}$$

$$a(t) = \sqrt{\frac{k}{2t}}$$

2.

$$m_1, m_2$$

$$\overline{a, T, F_j}$$



1. teleso:

$$m_1\vec{a}_1 = \vec{G}_1 + \vec{T}_1$$

2. teleso:

$$m_2\vec{a}_2 = \vec{G}_2 + \vec{T}_2$$

kladka:

$$0 = \vec{T}_1' + \vec{T}_2' + \vec{F}$$

$$-m_1 a_1 = -m_1 g + T_1$$
$$m_2 a_2 = -m_2 g + T_2$$
$$0 = -T_1' - T_2' + F$$

$$a_1 = a_2 = a$$
$$T_1 = T_1' = T_2 = T_2' = T$$

$$m_1 a = m_1 g - T$$
$$m_2 a = -m_2 g + T$$
$$F = 2T$$

$$(m_1+m_2)a = (m_1-m_2)g \Rightarrow \boxed{a = \frac{m_1-m_2}{m_1+m_2} g}$$

$$T = m_1(g-a) = m_1\left(g - \frac{m_1-m_2}{m_1+m_2} g\right) =$$

$$= m_1 g \left(1 - \frac{m_1-m_2}{m_1+m_2}\right) = m_1 g \frac{m_1+m_2-(m_1-m_2)}{m_1+m_2} =$$

$$= m_1 g \frac{m_1+m_2-m_1+m_2}{m_1+m_2}$$

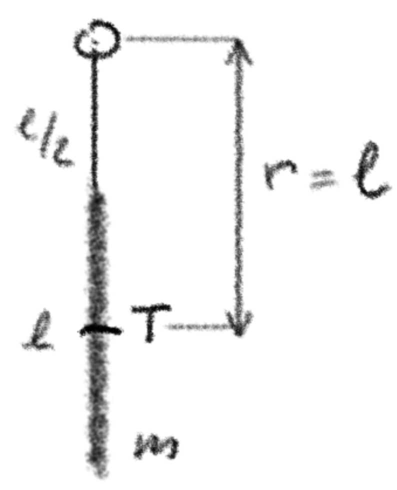
$$\boxed{T = \frac{2 m_1 m_2}{m_1+m_2} g}$$

$$F = 2T = \boxed{\frac{4 m_1 m_2}{m_1+m_2} g}$$

3. $\underline{m, \ell, \ell/2, J_T = \frac{1}{12} m \ell^2}$

$T;$

$$T = 2\pi \sqrt{\frac{J}{mgr}}$$



$r = l \qquad J = J_T + mr^2 = \frac{1}{12} ml^2 + ml^2$

$$J = \frac{13}{12} ml^2$$

$$T = 2\pi \sqrt{\frac{13ml^2}{12mgl}} \Rightarrow \boxed{T = 2\pi \sqrt{\frac{13l}{12g}}}$$

4. $\dfrac{m,\ k,\ x_0}{v_m;}$

1. spôsob → zákon zach. mechanickej energie

$E_p = \frac{1}{2} kx^2 \ ; \ E_k = \frac{1}{2} mv^2$

$$E_{k1} + E_{p1} = E_{k2} + E_{p2}$$

$E_{k1} = 0 \qquad E_{p1} = \frac{1}{2} k x_0^2 \qquad E_{k2} = \frac{1}{2} m v_m^2 \qquad E_k = 0$

$$\frac{1}{2} k x_0^2 = \frac{1}{2} m v_m^2 \Rightarrow \boxed{v_m = x_0 \sqrt{\frac{k}{m}}}$$

2. spôsob z kinematických vzťahov:
závažie na pružine je LHO

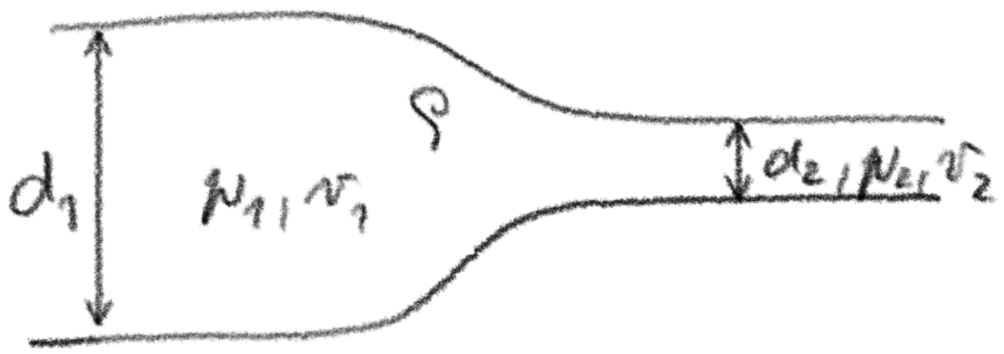
$x = x_0 \cos \omega t \qquad v = \frac{dx}{dt} = -x_0 \omega \sin \omega t$

kde $\omega = \sqrt{\frac{k}{m}} \qquad v = -v_m \sin \omega t$

$$\boxed{v_m = x_0 \omega = x_0 \sqrt{\frac{k}{m}}}$$

5. $d_1, p_1;\ d_2 < d_1, p_2, m, \rho$

$t;$



hmotnostný tok $\quad q_m = \rho S v = \frac{m}{t}$

$$t = \frac{m}{q_m} = \frac{m}{\rho S v}$$

$$S = \pi \frac{d^2}{4}$$

Rovnica spojitosti: $\quad S_1 v_1 = S_2 v_2$

$$\Rightarrow v_2 = \frac{S_1}{S_2} v_1 = \frac{d_1^2}{d_2^2} v_1$$

Bernoulliho rovnica: $\quad \frac{1}{2}\rho v_1^2 + p_1 = \frac{1}{2}\rho v_2^2 + p_2$

$$\frac{1}{2}\rho v_1^2 + p_1 = \frac{1}{2}\rho\left(\frac{d_1^2}{d_2^2} v_1\right)^2 + p_2$$

$$\rho v_1^2 \left(1 - \frac{d_1^4}{d_2^4}\right) = 2\,(p_2 - p_1)$$

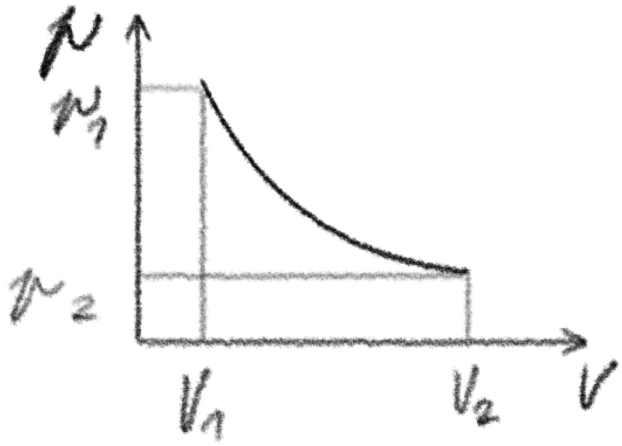
$$v_1 = \sqrt{\frac{2\,(p_2 - p_1)}{\rho\,(1 - d_1^4/d_2^4)}} = \sqrt{\frac{2\,(p_1 - p_2)}{\rho\,(d_1^4/d_2^4 - 1)}}$$

$$t = \frac{m}{\rho S_1 v_1} = \frac{m}{\pi d_1^2/4} \cdot \sqrt{\frac{d_1^4/d_2^4 - 1}{2\rho(p_1 - p_2)}}$$

$$\boxed{t = \frac{m}{\pi d_1^2 d_2^2}\sqrt{\frac{2(d_1^4 - d_2^4)}{\rho(p_1 - p_2)}}}$$

6. $V_1;\ V_2 = 2V_1,\ p_1,\ \varkappa$

$W;\ \Delta U$



$$p_1 V_1^{\varkappa} = p_2 V_2^{\varkappa}$$

$$W = \int_{V_1}^{2V_1} p\, dV$$

$$pV^{\varkappa} = p_1 V_1^{\varkappa} \Rightarrow p = \frac{p_1 V_1^{\varkappa}}{V^{\varkappa}}$$

$$W = \int_{V_1}^{2V_1} \frac{p_1 V_1^{\varkappa}}{V^{\varkappa}} dV = p_1 V_1^{\varkappa} \int_{V_1}^{V_2} V^{-\varkappa} dV =$$

$$= p_1 V_1^{\varkappa} \left[\frac{1}{1-\varkappa} V^{1-\varkappa}\right]_{V_1}^{2V_1} = p_1 V_1^{\varkappa} \frac{1}{1-\varkappa}\left(2^{1-\varkappa} V_1^{1-\varkappa} - V_1^{1-\varkappa}\right) =$$

$$= p_1 V_1^{\varkappa} \frac{V_1^{1-\varkappa}}{1-\varkappa}\left(2^{1-\varkappa} - 1\right)$$

$$\boxed{W = \frac{p_1 V_1}{\varkappa - 1}\left(1 - 2^{1-\varkappa}\right)}$$

Úlohu možno riešiť aj tak, že priamo využijeme vzťah pre prácu pri adiabatickom deji:

$$W = \frac{1}{\varkappa - 1}\left(p_1 V_1 - p_2 V_2\right)$$

$$p_1 V_1^{\varkappa} = p_2 V_2^{\varkappa} \Rightarrow p_2 V_2 = p_1 V_1^{\varkappa} V_2^{1-\varkappa} =$$

$$= p_1 V_1^{\varkappa}(2V_1)^{1-\varkappa} = 2^{1-\varkappa} p_1 V_1$$

$$W = \frac{1}{\varkappa - 1}\left(p_1 V_1 - 2^{1-\varkappa} p_1 V_1\right) = \frac{p_1 V_1}{\varkappa - 1}\left(1 - 2^{1-\varkappa}\right)$$

Adiabatický dej prebieha v izolovanej sústave, teda $\Delta Q = 0$

1. termodynamický zákon:

$$\Delta Q = \Delta U + W = 0$$

$$\Rightarrow \quad \Delta U = -W$$