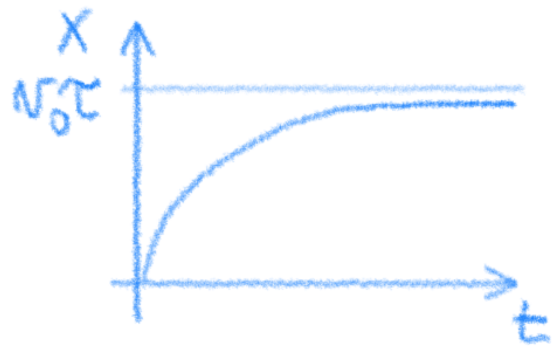
F1 LS 2025 písomka 1 riešenie

1. $x = v_0 \tau \left(1 - e^{-t/\tau}\right)$

v, a



$$x_{max} = \lim_{t\to\infty} v_0\tau\left(1 - e^{-t/\tau}\right) = v_0\tau$$

$$v = \frac{dx}{dt} = \frac{d}{dt} v_0\tau\left(1 - e^{-t/\tau}\right) = v_0 e^{-t/\tau}$$

$$a = \frac{dv}{dt} = \frac{d}{dt} v_0 e^{-t/\tau} = -\frac{v_0}{\tau} e^{-t/\tau}$$

$$a = -\frac{1}{\tau} v$$

2. a, R, s

ω, a_d

$s = \frac{1}{2} a t^2 \qquad \varphi = \frac{1}{2}\varepsilon t^2$

$v = at \qquad \omega = \varepsilon t$

$a = R\varepsilon$

$$t = \frac{\omega}{\varepsilon} = \frac{\omega}{a} R \rightarrow s = \frac{1}{2} a \cdot \frac{\omega^2 R^2}{a^2}$$

$$s = \frac{1}{2}\frac{R^2}{a}\omega^2$$

$$\omega = \frac{1}{R}\sqrt{2as}$$

$$a_d = R\omega^2 = R\frac{1}{R^2} 2as = \frac{2as}{R}$$

3. F, s, α, m

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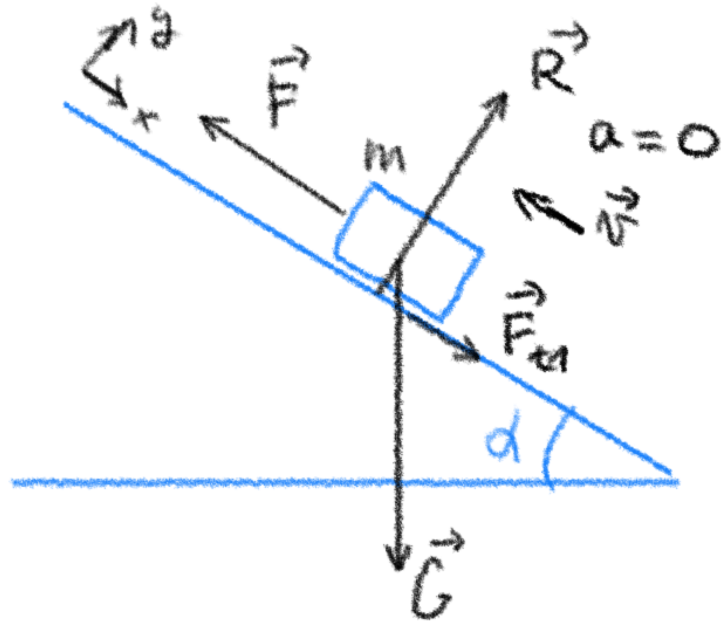
a

i) nahor



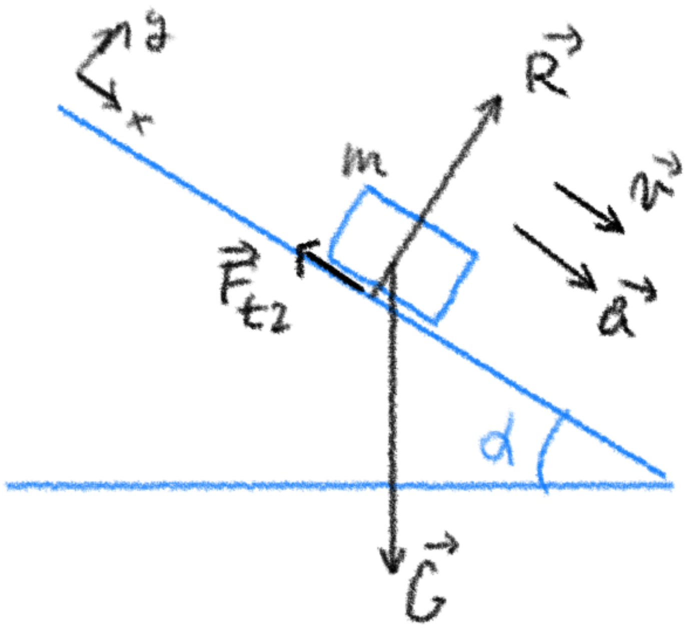
$$\vec{G}+\vec{R}+\vec{F}+\vec{F}_{t1}=0$$

$$x: \; mg\sin\alpha - F + F_{t1} = 0$$

$$y: \; -mg\cos\alpha + R = 0$$

$$F_{t1} = F - mg\sin\alpha$$

ii) nadol



$$\vec{G}+\vec{R}+\vec{F}_{t2}=m\vec{a}$$

$$x: \; mg\sin\alpha - F_{t2} = ma$$

$$y: \; -mg\cos\alpha + R = 0$$

$$a = g\sin\alpha - \frac{F_{t2}}{m}$$

$$F_{t1}=F_{t2} \Rightarrow a = g\sin\alpha - \frac{1}{m}\left(F - mg\sin\alpha\right)$$

$$\underline{\underline{a = 2g\sin\alpha - \frac{F}{m}}}$$

4. $s = \frac{1}{2} a t^2 \Rightarrow t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2s}{2g\sin\alpha - F/m}}$

$$v_{max} = at = a\sqrt{\frac{2s}{a}} = \sqrt{2as}$$

$$v_{max} = \sqrt{2s(2g\sin\alpha - F/m)}$$

$$F_t = \mu R \Rightarrow \mu = \frac{F_t}{R}$$

$$R = mg\cos\alpha \Rightarrow \mu = \frac{F - mg\sin\alpha}{mg\cos\alpha}$$

$$\mu = \frac{F}{mg\cos\alpha} - \operatorname{tg}\alpha$$

$$W_t = \vec{F_t} \cdot \vec{s} = -F_t s = (mg\sin\alpha - F)s$$

Možno riešiť aj pomocou zákona zachovania energie + práca trecej sily:

$$mgh + W_t = \frac{1}{2} m v_{max}^2$$

$$h = s\sin\alpha$$

$$mgs\sin\alpha + W_t = \frac{1}{2} m\left(2s(2g\sin\alpha - F/m)\right)$$

$$W_t = 2mgs\sin\alpha - mgs\sin\alpha - Fs$$

$$W_t = (mg\sin\alpha - F)\,s$$

práca je záporná, $F \geq mg\sin\alpha$ → to je podmienka rovnomerného pohybu smerom nahor.