Fyzika 1 LS 2024 Skúška 2 - Riešenie

1. $v(t) = \sqrt{2\ell t}$

$x(0) = x_0$

$x(t),\ a(t)$

$$x(t) = \int v(t)\,dt$$

$$x = \int \sqrt{2\ell t}\,dt =$$

$$= \sqrt{2\ell}\int t^{1/2}dt = \sqrt{2\ell}\,\frac{3}{2}t^{3/2} + C = 3\sqrt{\frac{1}{2}\ell t^3} + C$$

$$x(0) = 3\sqrt{\frac{1}{2}\ell\cdot 0^3} + C = x_0 \Rightarrow C = x_0$$

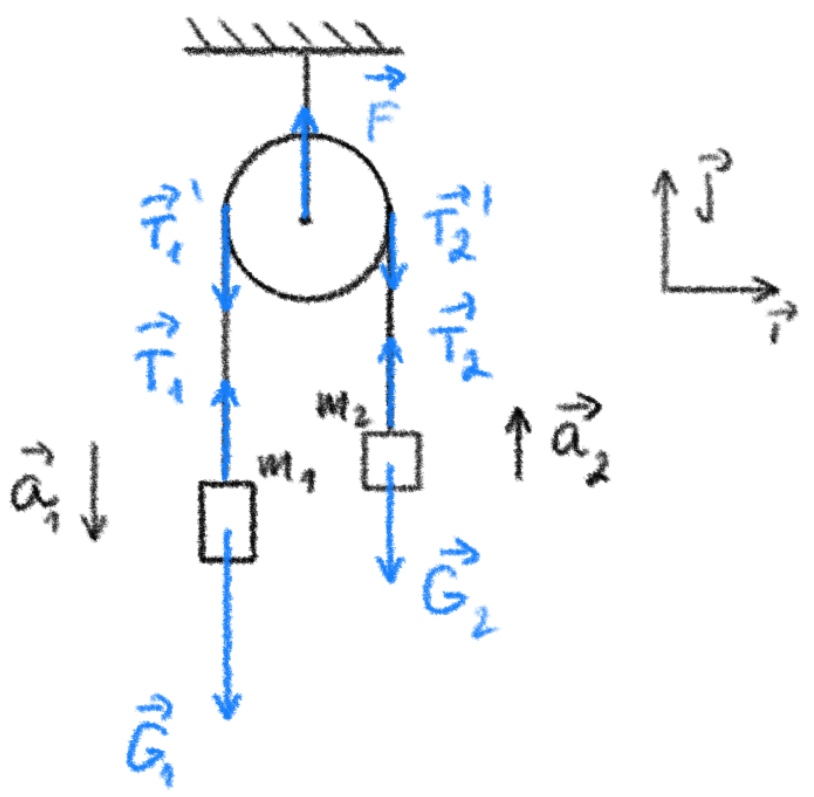
$$\boxed{x(t) = 3\sqrt{\frac{1}{2}\ell t^3} + x_0}$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\sqrt{2\ell t} = \sqrt{2\ell}\,\frac{d}{dt}t^{1/2} = \sqrt{2\ell}\,\frac{1}{2}t^{-1/2}$$

$$\boxed{a(t) = \sqrt{\frac{\ell}{2t}}}$$

2. m_1, m_2

$a,\ T,\ F_j$



1. teleso:

$$m_1\vec{a}_1 = \vec{G}_1 + \vec{T}_1$$

2. teleso:

$$m_2\vec{a}_2 = \vec{G}_2 + \vec{T}_2$$

kladka:

$$0 = \vec{T}_1' + \vec{T}_2' + \vec{F}$$

$$-m_1 a_1 = -m_1 g + T_1 \qquad a_1 = a_2 = a$$

$$m_2 a_2 = -m_2 g + T_2 \qquad T_1 = T_1' = T_2 = T_2' = T$$

$$0 = -T_1' - T_2' + F$$

$$m_1 a = m_1 g - T$$

$$m_2 a = -m_2 g + T$$

$$F = 2T$$

$$(m_1 + m_2) a = (m_1 - m_2) g \Rightarrow \boxed{a = \frac{m_1 - m_2}{m_1 + m_2} g}$$

$$T = m_1 (g - a) = m_1 \left(g - \frac{m_1 - m_2}{m_1 + m_2} g \right) =$$

$$= m_1 g \left(1 - \frac{m_1 - m_2}{m_1 + m_2} \right) = m_1 g \frac{m_1 + m_2 - (m_1 - m_2)}{m_1 + m_2} =$$

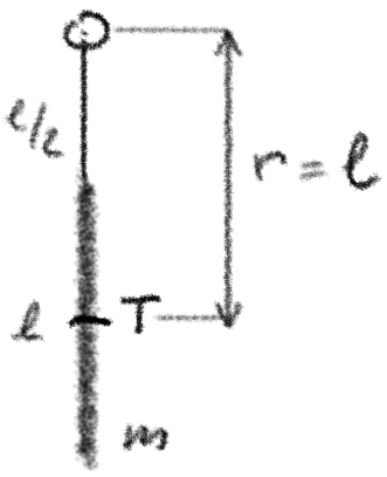
$$= m_1 g \frac{m_1 + m_2 - m_1 + m_2}{m_1 + m_2}$$

$$\boxed{T = \frac{2 m_1 m_2}{m_1 + m_2} g}$$

$$F = 2T = \boxed{\frac{4 m_1 m_2}{m_1 + m_2} g}$$

3. $m_1, \ell_1, \ell/2, J_T = \frac{1}{12} m \ell^2$

T;



$$T = 2\pi \sqrt{\frac{J}{mgr}}$$

$r = l$ $\qquad J = J_T + mr^2 = \frac{1}{12}ml^2 + ml^2$

$$J = \frac{13}{12}ml^2$$

$$T = 2\pi\sqrt{\frac{13ml^2}{12mgl}} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{13l}{12g}}}$$

4. $\quad \frac{m_1, k, x_0}{v_{mj}}$ $\qquad$ 1. spôsob → zákon zach. mechanickej energie

$$E_p = \frac{1}{2}kx^2 \; ; \; E_k = \frac{1}{2}mv^2$$

$$E_{k1} + E_{p1} = E_{k2} + E_{p2}$$

$$E_{k1} = 0 \qquad E_{p1} = \frac{1}{2}kx_0^2 \qquad E_{k2} = \frac{1}{2}mv_m^2 \qquad E_k = 0$$

$$\frac{1}{2}kx_0^2 = \frac{1}{2}mv_m^2 \Rightarrow \boxed{v_m = x_0\sqrt{\frac{k}{m}}}$$

2. spôsob z kinematických rovníc: závažie na pružine je LHO

$$x = x_0 \cos \omega t \qquad v = \frac{dx}{dt} = -x_0\omega \sin \omega t$$

kde $\omega = \sqrt{\frac{k}{m}}$ $\qquad v = -v_m \sin \omega t$

$$\boxed{v_m = x_0\omega = x_0\sqrt{\frac{k}{m}}}$$

5. d_1, ρ_1 ; $d_2 < d_1, \rho_2, m, \rho$

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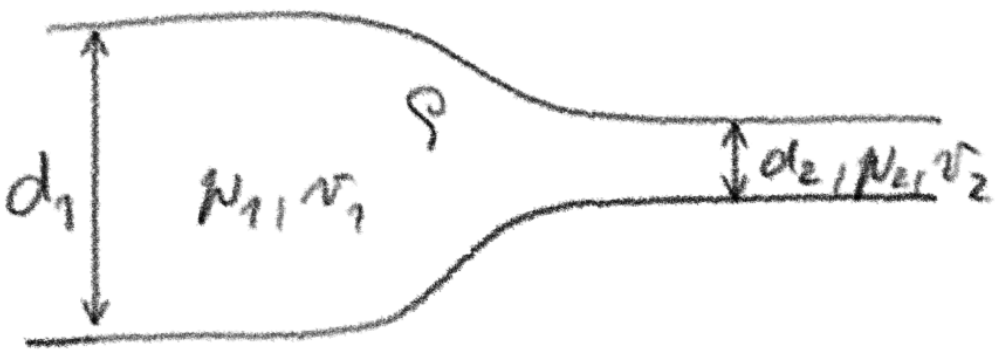
t ;



hmotnostný tok $q_m = \rho S v = \frac{m}{t}$

$$t = \frac{m}{q_m} = \frac{m}{\rho S v}$$

$S = \pi \frac{d^2}{4}$

Rovnica spojitosti: $S_1 v_1 = S_2 v_2$

$$\Rightarrow v_2 = \frac{S_1}{S_2} v_1 = \frac{d_1^2}{d_2^2} v_1$$

Bernoulliho rovnica: $\frac{1}{2}\rho v_1^2 + p_1 = \frac{1}{2}\rho v_2^2 + p_2$

$$\frac{1}{2}\rho v_1^2 + p_1 = \frac{1}{2}\rho \left(\frac{d_1^2}{d_2^2} v_1\right)^2 + p_2$$

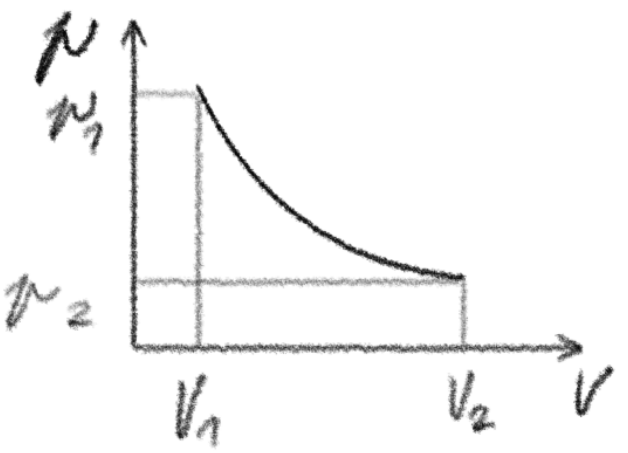
$$\rho v_1^2 \left(1 - \frac{d_1^4}{d_2^4}\right) = 2(p_2 - p_1)$$

$$v_1 = \frac{2(p_2 - p_1)}{\rho(1 - d_1^4/d_2^4)} = \frac{2(p_1 - p_2)}{\rho(d_1^4/d_2^4 - 1)}$$

$$t = \frac{m}{\varrho S_1 v_1} = \frac{m}{\pi d_1^2/4} \cdot \frac{d_1^4/d_2^4 - 1}{2(p_1 - p_2)}$$

$$t = \frac{2m\,(d_1^4/d_2^4 - 1)}{\pi d_1^2 (p_1 - p_2)}$$

6. V_1 ; $V_2 = 2V_1$) p_1) $\varkappa$



W ; ΔU

$$p_1 V_1^{\varkappa} = p_2 V_2^{\varkappa}$$

$$W = \int_{V_1}^{2V_1} p\,dV$$

$$pV^{\varkappa} = p_1 V_1^{\varkappa} \Rightarrow p = \frac{p_1 V_1^{\varkappa}}{V^{\varkappa}}$$

$$W = \int_{V_1}^{2V_1} \frac{p_1 V_1^{\varkappa}}{V^{\varkappa}}\,dV = p_1 V_1^{\varkappa} \int_{V_1}^{V_2} V^{-\varkappa}\,dV =$$

$$= p_1 V_1^{\varkappa} \left[\frac{1}{1-\varkappa} V^{1-\varkappa}\right]_{V_1}^{2V_1} = p_1 V_1^{\varkappa} \frac{1}{1-\varkappa}\left(2^{1-\varkappa} V_1^{1-\varkappa} - V_1^{1-\varkappa}\right) =$$

$$= p_1 V_1^{\varkappa} \frac{V_1^{1-\varkappa}}{1-\varkappa}\left(2^{1-\varkappa} - 1\right)$$

$$W = \frac{p_1 V_1}{\varkappa - 1}\left(1 - 2^{1-\varkappa}\right)$$

Úlohu možno riešiť aj tak, že priamo využijeme vzťah pre prácu pri adiabatickom deji:

$$W = \frac{1}{\varkappa - 1}\left(p_1 V_1 - p_2 V_2\right)$$

$$p_1 V_1^{\varkappa} = p_2 V_2^{\varkappa} \Rightarrow p_2 V_2 = p_1 V_1^{\varkappa} V_2^{1-\varkappa} =$$

$$= p_1 V_1^{\varkappa} (2V_1)^{1-\varkappa} = 2^{1-\varkappa} p_1 V_1$$

$$W = \frac{1}{\varkappa - 1}\left(p_1 V_1 - 2^{1-\varkappa} p_1 V_1\right) = \frac{p_1 V_1}{\varkappa - 1}\left(1 - 2^{1-\varkappa}\right)$$

Adiabatický dej prebieha v izolovanej sústave, teda $\Delta Q = 0$

1. termodynamický zákon:

$$\Delta Q = \Delta U + W = 0$$

$$\rightarrow \quad \Delta U = -W$$