F1 LS 2024, skúška 1, úlohy - riešenie

1.

$$v(t) = v_0 e^{-bt}$$
$$x(0) = 0$$

$x(t)$; $a(t)$

$$a(t) = \frac{dv}{dt} = -v_0 b e^{-bt}$$

$$x(t) = \int v(t)\,dt = \int v_0 e^{-bt} dt = -\frac{1}{b} v_0 e^{-bt} + C$$

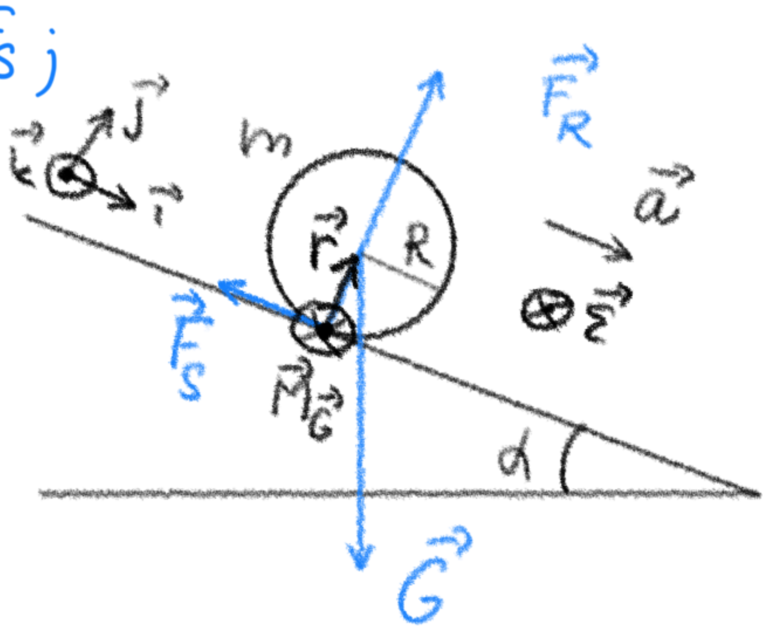
$$x(0) = -\frac{v_0}{b} e^0 + C = C - \frac{v_0}{b} = 0$$

$$C = \frac{v_0}{b}$$

$$x(t) = -\frac{v_0}{b} e^{-bt} + \frac{v_0}{b} = \frac{v_0}{b}\left(1 - e^{-bt}\right)$$

2. $m, R, J_T = \frac{1}{2} m R^2, \alpha$

F_S; j



$$\vec{M}_R = \vec{M}_S = \vec{0}$$

1. pohybová rovnica

$$m\vec{a} = \vec{G} + \vec{F}_R + \vec{F}_S$$

2. pohybová rovnica

$$J\vec{\varepsilon} = \vec{M}_G + \vec{M}_R + \vec{M}_S$$

rotácia okolo bodu dotyku $\Rightarrow J = J_T + mR^2$

$$J = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2$$

$$\vec{M}_S = \vec{0} \quad ; \quad \vec{M}_R = \vec{0} \quad ; \quad \vec{M}_G = \vec{r} \times \vec{G}$$

$$M_G = R\,mg\,\sin(\pi - \alpha) = mgR\,(\underbrace{\sin\pi}_{=0}\cos\alpha - \sin\alpha\underbrace{\cos\pi}_{=-1})$$

$$M_G = mgR\sin\alpha$$

$$\vec{G} = mg\,(\vec{i}\sin\alpha - \vec{j}\cos\alpha)$$

$$\vec{F}_R = F_R\,\vec{j} \qquad \vec{F}_S = -F_S\,\vec{i}$$

$$\vec{M}_G = -(mgR\sin\alpha)\,\vec{k}$$

$$\vec{a} = a\,\vec{i} \qquad \vec{\varepsilon} = -\varepsilon\,\vec{k} = -\frac{a}{R}\vec{k}$$

$$\left.\begin{aligned} x:&\quad ma = mg\sin\alpha - F_S \\ y:&\quad 0 = -mg\cos\alpha + F_R \end{aligned}\right\}\ \text{1. pohybová rovnica}$$

$$z:\ -\frac{3}{2}mR^2\frac{a}{R} = -mgR\sin\alpha \quad \}\ \text{2. pohybová rovnica}$$

$$a = \frac{2}{3}g\sin\alpha$$

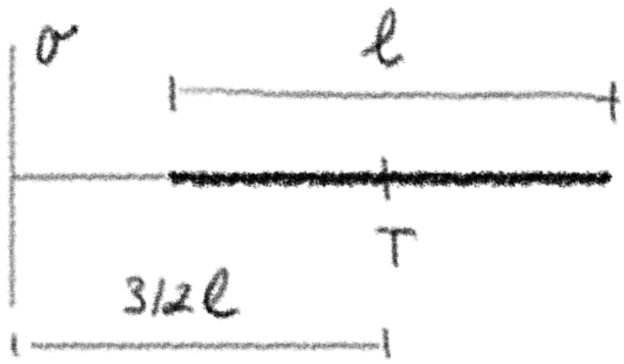
$$m \cdot \frac{2}{3}g\sin\alpha = mg\sin\alpha - F_S$$

$$F_S = mg\sin\alpha - \frac{2}{3}mg\sin\alpha$$

$$F_S = \frac{1}{3}mg\sin\alpha$$

3. $m, l, \omega, x = \frac{3}{2} l; \omega_1 = \frac{1}{2}\omega, M; J_T = \frac{1}{12} m l^2$

$J; E_k; \Delta t$



$$J = J_T + m\left(\frac{3}{2} l\right)^2$$

$$J = \frac{1}{12} m l^2 + \frac{9}{4} m l^2$$

$$J = \frac{7}{3} m l^2$$

$$E_k = \frac{1}{2} J \omega^2 = \frac{1}{2}\,\frac{7}{3} m l^2 \omega^2 = \frac{7}{6} m l^2 \omega^2$$

Konštantný moment sily ⇒ rovnomerne spomalený rotačný pohyb:

$$\omega_1 = \omega - \varepsilon \Delta t$$

$$\Delta t = \frac{\omega - \omega_1}{\varepsilon} = \frac{\omega - \frac{1}{2}\omega}{\varepsilon} = \frac{1}{2}\,\frac{\omega}{\varepsilon}$$

$$J\varepsilon = M \Rightarrow \varepsilon = \frac{M}{J} = \frac{3}{7}\,\frac{M}{m l^2}$$

$$\Delta t = \frac{1}{2}\omega \frac{7}{3}\,\frac{m l^2}{M} = \frac{7}{6}\,\frac{m \omega l^2}{M}$$

4. $y(t) = A \sin(\omega t + \varphi)$

$y_1;$

$E_p = \frac{1}{2} k y^2$ $\qquad$ $E_k = \frac{1}{2} m v^2$

$$v = \frac{dy}{dt} = A\omega \cos(\omega t + \varphi)$$

$$E_p = \frac{1}{2} k A^2 \sin^2(\omega t + \varphi)$$

$$E_k = \frac{1}{2} m A^2 \omega^2 \cos^2(\omega t + \varphi)$$

$$\omega^2 = \frac{k}{m} \Rightarrow \quad m\omega^2 = k$$

$$E_k = \frac{1}{2} k A^2 \cos^2(\omega t + \varphi)$$

$$E_p = E_k$$

$$\sin^2(\omega t + \varphi) = \cos^2(\omega t + \varphi)$$

to je v okamihoch $\omega t + \varphi = \pm \frac{\pi}{4}; \pm \frac{3}{4}\pi; \pm \frac{5}{4}\pi$

vtedy $\sin(\omega t + \varphi) = \pm \frac{\sqrt{2}}{2}$

$$y_1 = \pm A \frac{\sqrt{2}}{2} = \pm \frac{A}{\sqrt{2}}$$

$$|y_1| = \frac{A}{\sqrt{2}}$$

5. $\rho_1, m, \rho_2 < \rho_1$

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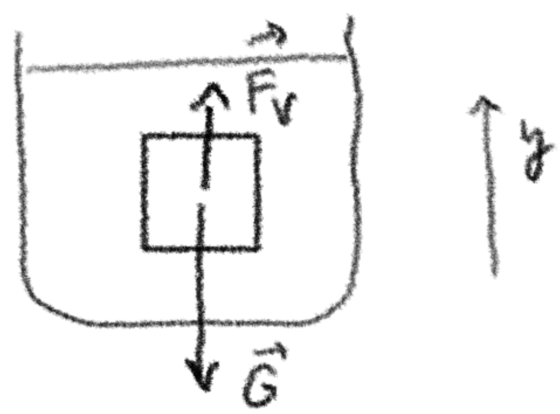
m'



Silomer meria celkovú silu pôsobiacu na teleso

$$m = \frac{G}{g} \qquad m' = \frac{F}{g}$$

$$\vec{F} = \vec{G} + \vec{F_v}$$

$$y: \; -F = -mg + F_v \Rightarrow F = mg - F_v$$

$$F_v = \rho_2 g V \qquad \rho_1 = \frac{m}{V} \Rightarrow V = \frac{m}{\rho_1}$$

$$F_v = \rho_2 g \frac{m}{\rho_1} = \frac{\rho_2}{\rho_1} mg$$

$$F = mg - \frac{\rho_2}{\rho_1} mg = mg\left(1 - \rho_2/\rho_1\right)$$

$$m' = \frac{F}{g} = m\left(1 - \rho_2/\rho_1\right)$$

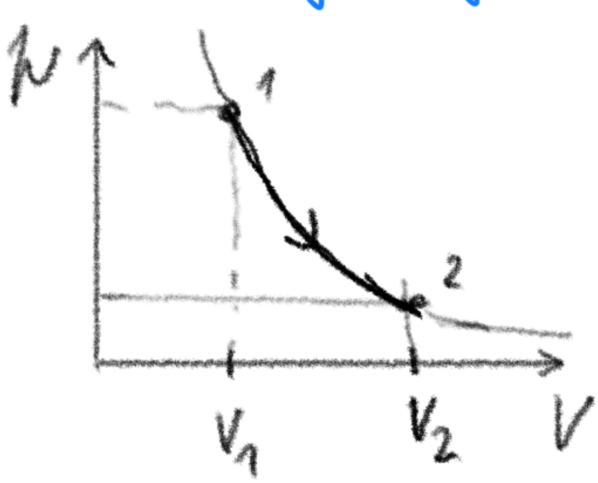
6. $n, T; \; V_2/V_1 = 2$

$W; Q$

$pV = konst$

$pV = nRT$

izotermický dej $\Rightarrow \Delta U = 0 \Rightarrow W = Q$



$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{nRT}{V} dV =$$

$$= nRT \int_{V_1}^{V_2} \frac{dV}{V} = nRT \ln V_2/V_1$$

$$W = nRT \ln 2 = Q$$