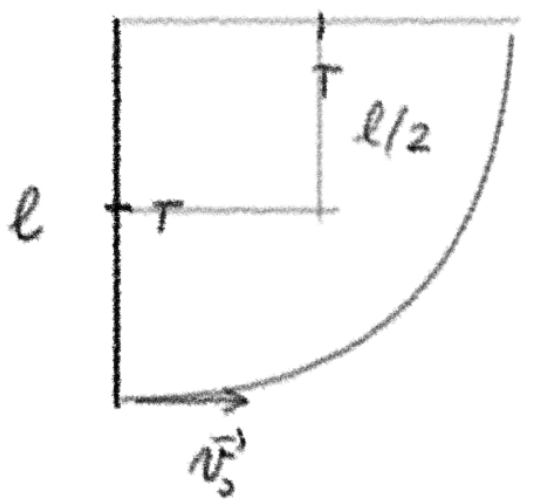
F1 – LS 2024, písomka 2, riešenie

1. ℓ, $J_T = \frac{1}{12} m\ell^2$

v_0;



$$\frac{1}{2} J \omega_0^2 = mg \frac{\ell}{2}$$

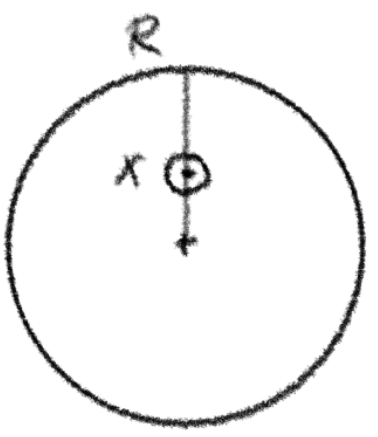
$$\omega_0 = \frac{v_0}{\ell}, \quad J = \frac{1}{12} m\ell^2 + m\left(\frac{\ell}{2}\right)^2 = \frac{1}{3} m\ell^2$$

$$\frac{1}{2} \frac{1}{3} m\ell^2 \frac{v_0^2}{\ell^2} = mg \frac{\ell}{2} \Rightarrow v_0^2 = 3g\ell$$

$$v_0 = \sqrt{3g\ell}$$

2. R, $J_T = \frac{1}{2} mR^2$

x_j



$$T = 2\pi \sqrt{\frac{J}{mgr}}$$

$$J = J_T + mx^2 = \frac{1}{2} m(R^2 + 2x^2)$$

$$r = x$$

$$T = 2\pi \sqrt{\frac{m(R^2+2x^2)}{2mgx}} = 2\pi \sqrt{\frac{R^2+2x^2}{2gx}}$$

$$\frac{dT}{dx} = 0$$

$$\frac{d}{dx} 2\pi \sqrt{\frac{R^2+2x^2}{2gx}} = 2\pi \frac{1}{2} \sqrt{\frac{2gx}{R^2+2x^2}} \cdot \frac{4x \cdot 2gx - (R^2+2x^2)2g}{4g^2x^2} =$$

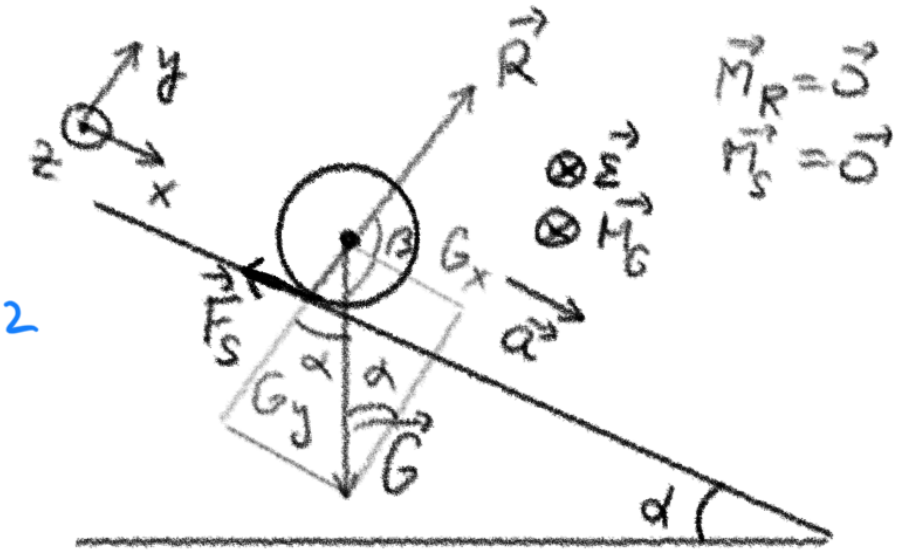
$$= \pi \sqrt{\frac{2gx}{R^2+2x^2}} \frac{2x^2 - R^2}{2gx^2} = 0$$

$$2x^2 - R^2 = 0 \Rightarrow x^2 = \frac{R^2}{2}$$

$$x = \frac{R}{\sqrt{2}} \qquad (x \geq 0)$$

3. R, m, α

$\omega, \quad J_T = \frac{2}{5} m R^2$



1. Pohybová rovnica
$m\vec{a} = \vec{G} + \vec{R} + \vec{F}_S$

2. Pohybová rovnica
$J\vec{\varepsilon} = \vec{M}_S + \vec{M}_G + \vec{M}_R$

Rotácia voči osi prechádzajúcej bodom dotyku. Vtedy 1. rovnicu nepotrebujeme:

$$J = J_T + mR^2 = \frac{7}{5} mR^2 \qquad \vec{M}_S = \vec{M}_R = \vec{0}$$

$$|\vec{M}_G| = |\vec{r}_G \times \vec{G}| = Rmg\sin\beta = Rmg\sin(\pi - \alpha)$$

$$M_G = Rmg\sin\alpha$$

2. pohybová rovnica:

$$-J\varepsilon = -Rmg\sin\alpha \qquad \varepsilon = \frac{a}{R}$$

$$\frac{7}{5}mR^2\frac{a}{R} = Rmg\sin\alpha$$

$$a = \frac{5}{7}g\sin\alpha$$

4. $\frac{N}{n_j}$

$$A_0 e^{-bNT} = \frac{1}{2}A_0$$

$$-bNT = \ln\frac{1}{2}$$

$$bNT = \ln 2$$

$$b = \frac{\ln 2}{NT}$$

$$E_0 e^{-2bnT} = \frac{1}{10}E_0 \Rightarrow -2bnT = \ln\frac{1}{10}$$

$$2bnT = \ln 10 \Rightarrow n = \frac{\ln 10}{2bT} = \frac{NT}{2T}\frac{\ln 10}{\ln 2}$$

$$n = \frac{N}{\log 4}$$